31/3/2018

الزوغي

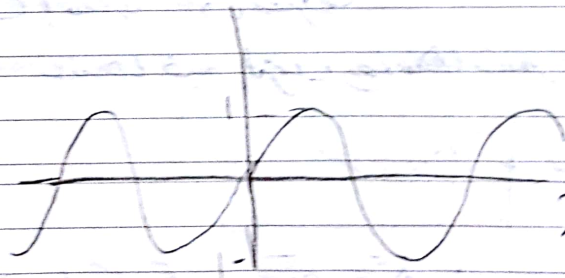
Signals :- الإشارات

→ Classifications of signals :-

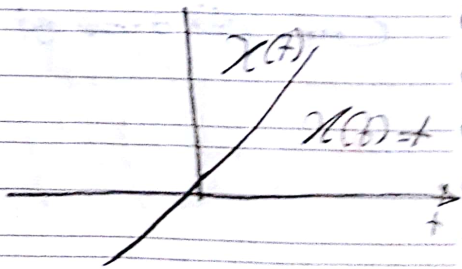
[1] deterministic or Random
محسوبة عشوائية

← اي اشارة يمكن التعبير عنها بمعادلة رياضية تسمى

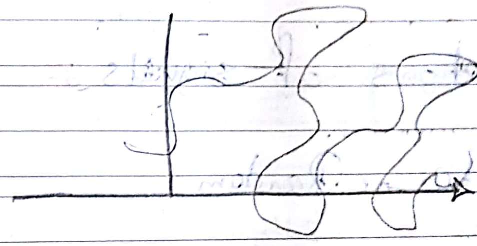
(Deterministic signal)



$x(t) = \sin(t)$

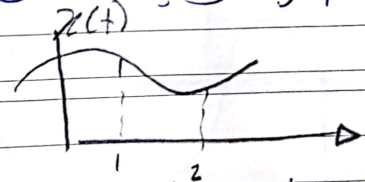


اي اشارة لا يمكن التعبير عنها بمعادلة رياضية تسمى إشارة عشوائية (random signal)

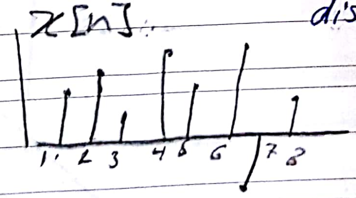


[2] Continuous time or Discrete, time
مستمرة متقطعة

اي إشارة تكون معرفة عندها قيم الزمن وما بينها تسمى Continuous



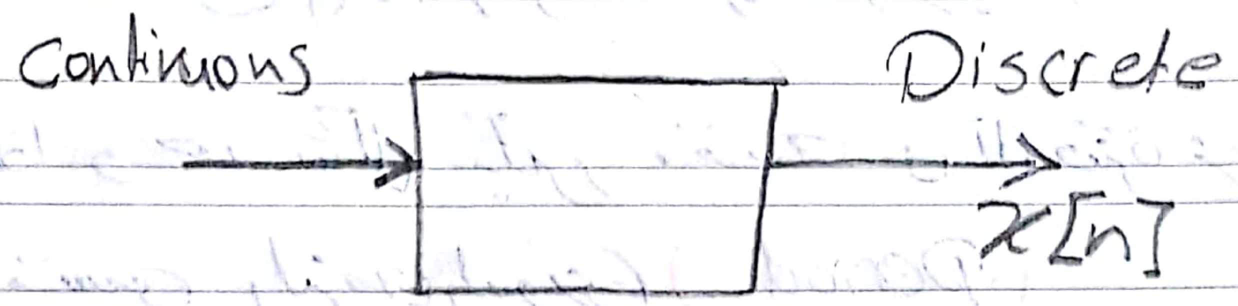
اي إشارة تكون معرفة عندها قيم محدودة فقط تسمى إشارة متقطعة discrete

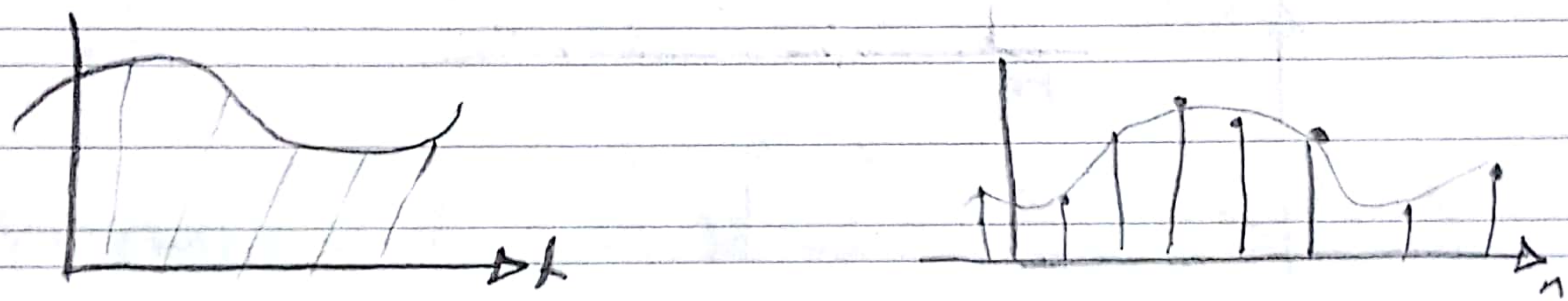


$n, k\epsilon$ $k = 1, 2, 3, \ldots$

يمكن التحويل من (Contionuous) إلى (Discrete)

عن طريق (Sampler)





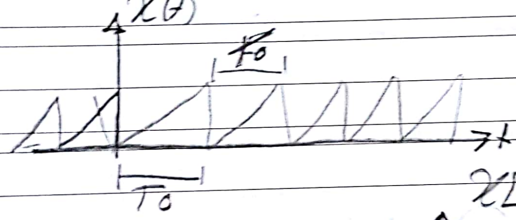
4 Periodic or Nonperiodic (aperiodic)

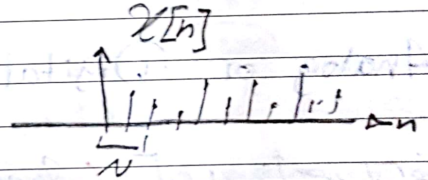
دورية — غير دورية

← تسمى الإشارة وإشارة دورية (Periodic)

إذا وجد شكل يكرر نفسه في كل فترة زمنية محددة

تسمى الزمن الدوري (Period)





← رياضياً تسمى الإشارة إشارة دورية إذا كان

$x(t) = x(t \pm nT_0) \quad \forall t$

$\forall$ → For all

$n = 1, 2, 3, 4 \ldots$

T_0 is Called Fundamental period

← تسمى الإشارة إشارة غير دورية إذا كان

$x(t) \neq x(t \pm nT_0)$

5 Energy or Power

طاقة ... قدرة

normalized

$$P = \frac{dw}{dt} \longrightarrow dw = P\,dt$$

$$dw = \frac{V^2}{R}\,dt$$

$$W = \int \frac{V^2}{R}\,dt \qquad R = 1\Omega$$

normalized Energy

$$W = \int_{\infty} V^2\,dt$$

$$E_X = \int_{-\infty}^{\infty} x^2(t)\,dt \qquad \text{normalized Energy}$$

تسمى بإشارة طاقة (Energy signal)

$$0 < E_X < \infty$$

- تسمى ال إشارة إشارة قدرة (Power Signal)
إذا كان $0 < P_x < \infty$

- هناك إشارات ليست إشارات طاقة
ولا إشارات قدرة
neigther Energy nor power

* ملاحظات:

[1] إذا كانت ال إشارة محدودة في الزمن فإنها إشارة طاقة (Energy Signal).

[2] إذا كانت ال إشارة قيمتها صفر عندما $-\infty$ و ∞ فإنها إشارة طاقة (Energy).

Ex: Find the Energy for $x(t) = e^{-at}$, $a>0, t>0$

$$E_x = \int_{-\infty}^{\infty} x^2(t)\, dt$$

$$E_x = \int_0^{\infty} (e^{-at})^2 \, dt = \int_0^{\infty} e^{-2at} \, dt$$

$$= \left[\frac{e^{-2at}}{-2a} \right]_0^{\infty} = -\frac{1}{2a} \left[e^{-\infty} - e^{0} \right]$$

$$= -\frac{1}{2a} [0 - 1] = \frac{1}{2a}$$

$$e^{0} = 1, \quad e^{\infty} \to \infty, \quad e^{-\infty} = 0$$

$\boxed{ع}$ اي اشارة دورية هي اشارة قدرة (power signal) ~~Ex~~*

والعكس غير صحيح

$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t) \, dt$$

Ex: Find the Power for $x(t)$

$$x = A\cos(\omega t)$$

$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} [A\cos(\omega t)]^2 \, dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \frac{A^2}{2} \left[1 + \cos 2\omega t\right] dt$$

$$= \lim_{T\to\infty} \frac{A^2}{2T} \int_{-T/2}^{T/2} \left[1 + \underbrace{\cos(2\omega t)}_{0}\right] dt$$

$$\lim_{T\to\infty} \frac{A^2}{2T} \int_{-T/2}^{T/2} dt$$

$$P_x = \lim_{T\to\infty} \frac{A^2}{2\cancel{T}} \cancel{T} = \lim \frac{A^2}{2} = \frac{A^2}{2}$$

[6] Even or odd:

زوجية   فردية

(Even) : تسمى الإشارة إشارة زوجية :
اذا كانت متماثلة حول محور الصادات

رياضياً اذا كان

$$x(t) = x(-t)$$

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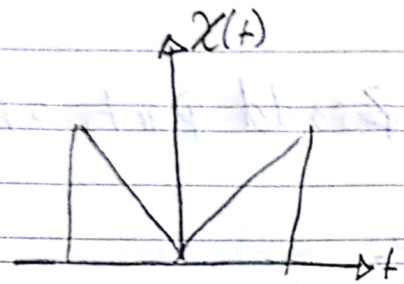
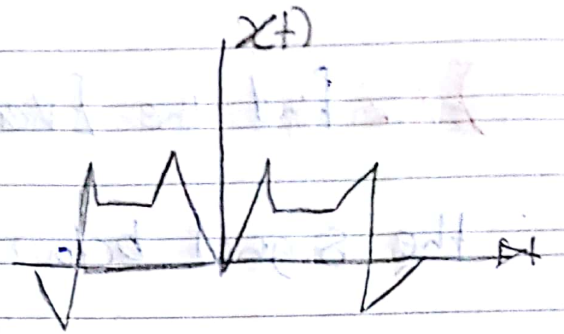
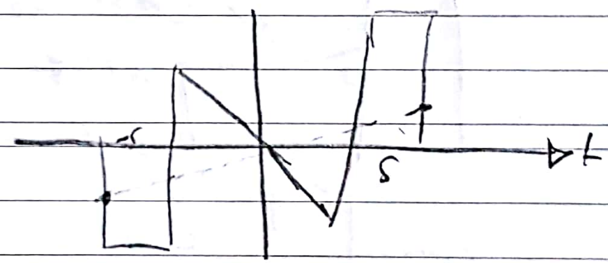




- تسمى الإشارة إشارة فردية اذا كانت متماثلة حول نقطة الأصل
- رياضياً اذا كانت

$$x(t) = -x(-t)$$



- هناك اشارات ليست فردية ولا زوجية.
- اذا كانت الإشارة ليست فردية ولا زوجية فإنه يمكن كتابتها على صورة

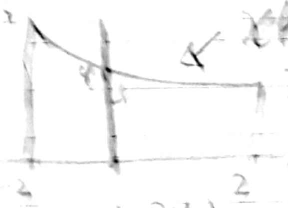
$$x(t) = x_e(t) + x_o(t)$$

$$x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right]$$

$$x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right]$$

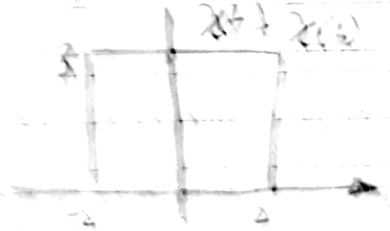
11. Find the Even & odd parts of the signal below:

$x(t) = 0.5t + 4$

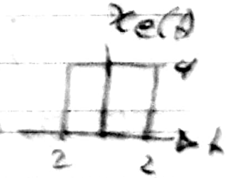


$x(-t) = -0.5t + 4$

$x(t) + x(-t) = 8$

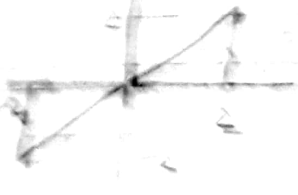


$\Rightarrow$



$x(t) - x(-t) = 0.5t + 4 - (-0.5t + 4) = t$

$x(t) - x(-t) = t$



$\Rightarrow$

* Basic operations :

[1] Time Shefting إزاحة بالزمن

$a > 0$

$x(t-a) \rightarrow x(t)$ shifted to Reght →

$x(t+a) \rightarrow x(t)$ Shifted to left ←

[2] Time Scaling تغيير زمني

$a > 0$

$x(at) \rightarrow x(t)$

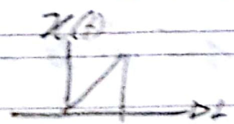
انضغاط
Compression
$a > 1$

توسع
Expansion
$1 > a > 0$

[3] Time inversion

$x(-t) \rightarrow x(t)$ Inverted

تنعكس حول محور الصادات



4 Amplitude shifted

الإزاحة السعوية

$a + x(t) \longrightarrow x(t)$ shifted up ↑

$x(t) - a \longrightarrow x(t)$ shifted down ↓

5 Amplitude scaling

تقييس سعوي

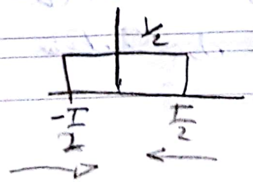
$A\,x(t) \longrightarrow A$ multiplied by $x(t)$

A مكانه $x(t)$ (Amplitude) ضرب سعة

Time shifting → Time scaling → Time inversion → Amplitude scaling → Amplitude shifting

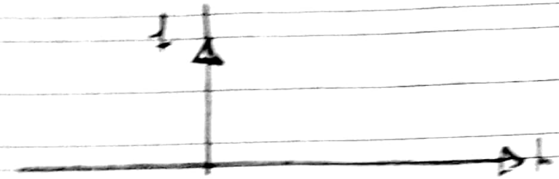
⇒ Important signals :-

1 Impulse signal إشارة النبضة



$I = \frac{1}{T} = 1 \leftarrow$ Area $= 1$

$x(t) = \delta(t)$



$$\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & t \neq 0 \end{cases}$$

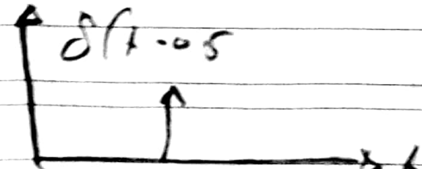
$$\delta(\boxed{\ }) = \begin{cases} 1 & \boxed{\ } = 0 \\ 0 & \boxed{\ } \neq 0 \end{cases}$$

* Properties of $\delta(t)$

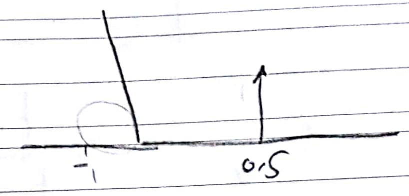
$\boxed{1}$ $\int_{-\infty}^{\infty} \delta(t)\, dt = 1$

$\boxed{2}$ $\int_{a}^{b} \delta(t-k)\, dt \begin{cases} 1 & a < k < b \\ 0 & k < a \text{ or } k > b \end{cases}$

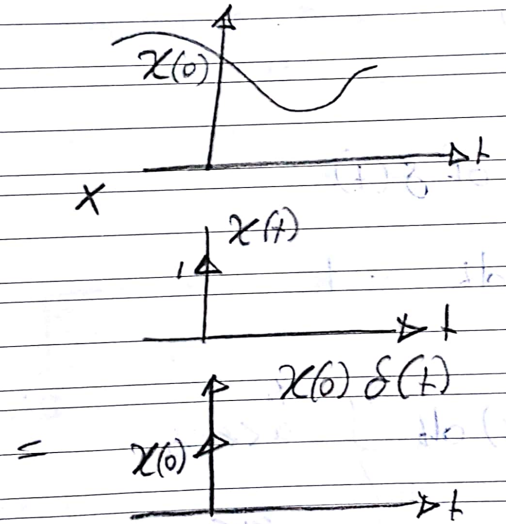
Ex: $\int_{0}^{1} \delta(t-0.5)\, dt = 1$



$$\int_{-1}^{0} \delta(t-0.5)\,dt = 0$$



[3] $x(t)\,\delta(t) = x(0)\,\delta(t)$



[4] $x(t)\,\delta(t-k) = x(k)\,\delta(t-k)$

$t-k=0$

$t=k$

(5) $\int_{-\infty}^{+\infty} x(t)\,\delta(t)\,dt = x(0)$

$\int_{-\infty}^{+\infty} x(t)\,\delta(t)\,dt = \int_{-\infty}^{+\infty} x(0)\,\delta(t)\,dt$

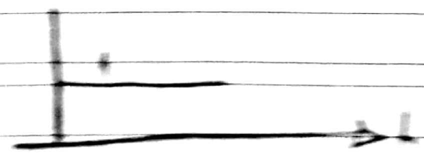
$x(0) \int_{-\infty}^{+\infty} \delta(t)\,dt = x(0)$

(6) $\int_{-\infty}^{+\infty} x(t)\,\delta(t-t_0)\,dt = x(t_0)$

$\int_{-\infty}^{+\infty} x(t)\,\delta(t-t_0)\,dt = \int_{-\infty}^{+\infty} x(t_0)\,\delta(t-t_0)\,dt$

$= x(t_0) \int_{-\infty}^{+\infty} \delta(t-t_0)\,dt = x(t_0)$

(7) unit step [illegible]

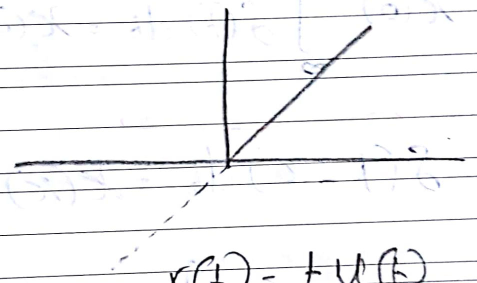


$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$

$$u(\boxed{}) = \begin{cases} 1 & \boxed{} \geq 0 \\ 0 & \boxed{} < 0 \end{cases}$$

$\boxed{3}$ ramp signal اشارة منحدر

$x(t) = r(t)$



$r(t) = t\,u(t)$

$r(\boxed{}) = \boxed{}\, u(\boxed{})$

Ex: $r(t-5) = (t-5)\, u(t-5)$

$\boxed{1}$ $r(t)$

اكمل

$\boxed{2}$ $r(t)$

اكمل

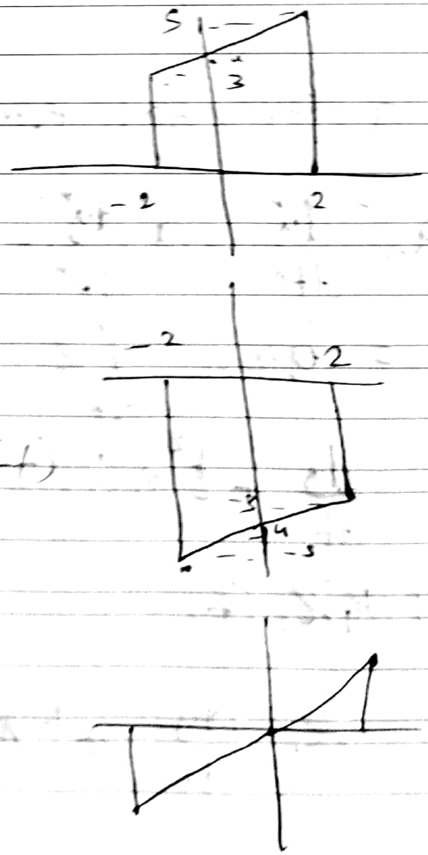
$$\delta(t) \underset{\text{تفاضل}}{\overset{\text{تكامل}}{\rightleftarrows}} u(t) \underset{\text{تفاضل}}{\overset{\text{تكامل}}{\rightleftarrows}} r(t)$$

$x(t) + x(-t) \longrightarrow$ even

$x(t) + (-x(-t)) \longrightarrow$ odd

$x(t)$

$x(-t)$ $\quad$ $-x(-t)$



$-x(-t)$

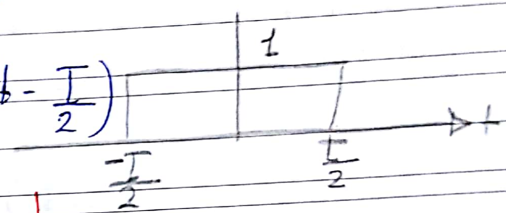
1/4/2018

مكرري

[4] rectangular function :

$$x(t) = \text{rect}\left(\frac{t}{T}\right)$$

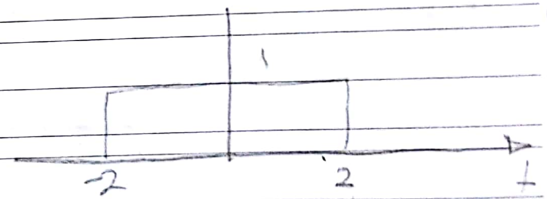
$$\text{rect}\left(\frac{t}{T}\right) = u\left(t+\frac{T}{2}\right) - u\left(t-\frac{T}{2}\right)$$



Ex: sketch the following signals

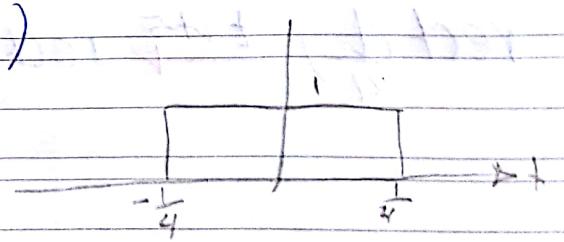
[1] $x(t) = \text{rect}\left(\frac{t}{4}\right)$

$\text{rect}\left(\frac{t}{4}\right)$

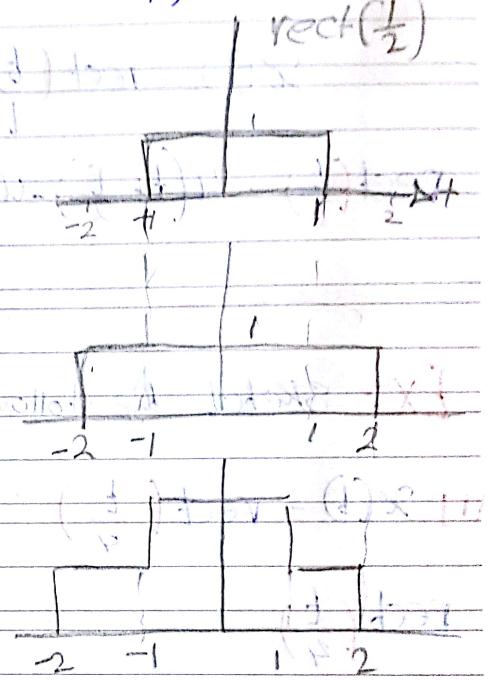


[2] $x(t) = \text{rect}\,(2t)$

$= \text{rect}\left(\frac{t}{\frac{1}{2}}\right)$

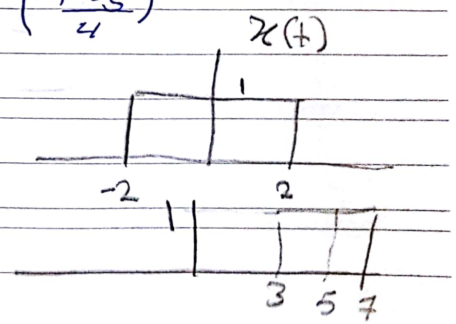


[3] $x(t) = \text{rect}\left(\frac{t}{2}\right) + \text{rect}\left(\frac{t}{4}\right)$



[4] $x(t) = \text{rect}\left(\frac{t-5}{4}\right)$

$\text{rect}\left(\frac{t}{4}\right) \xrightarrow{t \to t-5} \text{rect}\left(\frac{t-5}{4}\right)$



$\boxed{5}$ $x(t) = 3\,\text{rect}\left(3t - \frac{1}{5}\right)$

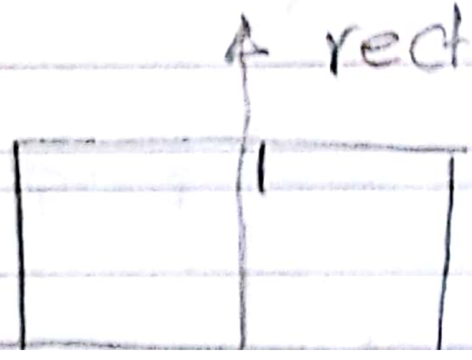
$$3t - \frac{1}{5} = \frac{15-1}{5}$$

$$x(t) = 3\,\text{rect}\left(\frac{15-1}{5}\right)$$

$$\text{rect}\left(\frac{t}{5}\right) \xrightarrow{t \to t-1} \text{rect}\left(\frac{t-1}{5}\right)$$

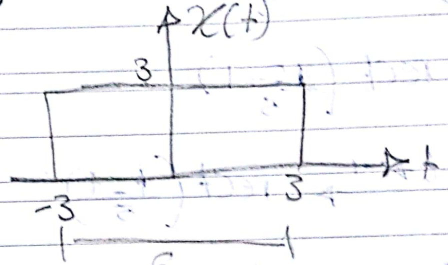
$$\text{rect}\left(\frac{15t-1}{5}\right) \xleftarrow{t \to 15t}$$

$\text{rect}\left(\frac{t}{5}\right)$



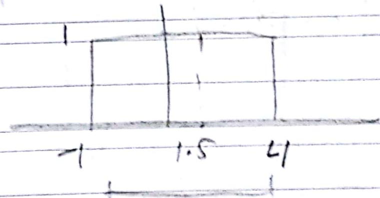
6) Express the following signal in terms of rectangular signal

A)



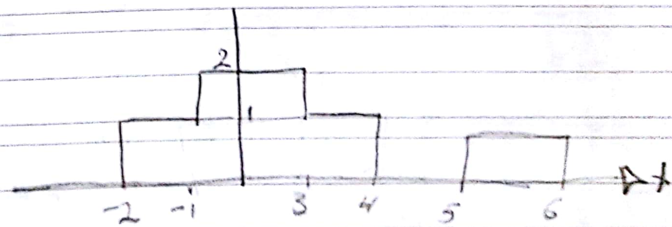
$x(t) = 3\,\mathrm{rect}\left(\frac{t}{6}\right)$

B)

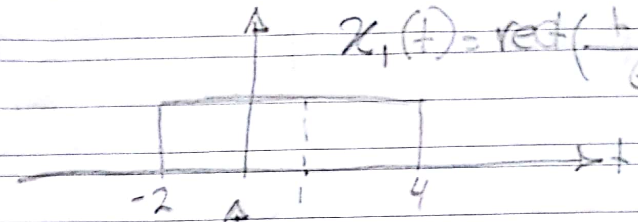


$x(t) = \mathrm{rect}\left(\frac{t-1.5}{5}\right)$

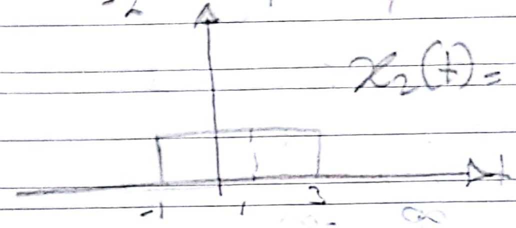
C)

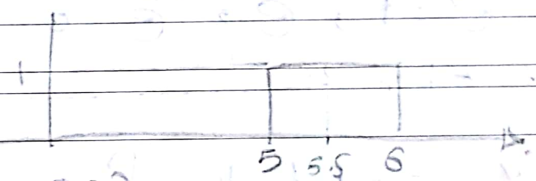


$x_1(t) = \mathrm{rect}\left(\frac{t-1}{6}\right)$



$x_2(t) = \mathrm{rect}\left(\frac{t-1}{6}\right)$





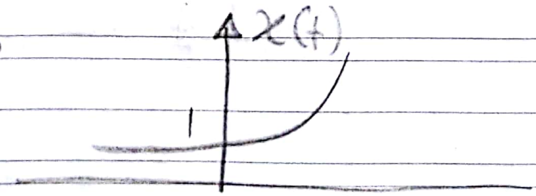
$x_3(t) = \mathrm{rect}(t-5.5)$

$x(t) = \mathrm{rect}\left(\frac{t-1}{6}\right) + \mathrm{rect}\left(\frac{t-1}{4}\right) + \mathrm{rect}(t-5.5)$

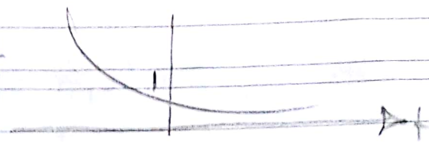
$$\mathrm{rect}\left(\frac{t}{\tau}\right) = \begin{bmatrix} 1 & -\frac{\tau}{2} \le t \le \frac{\tau}{2} \\ 0 & o.w \end{bmatrix}$$

5 Exponential Function

$x(t) = e^{t}$



$x(t)=e^{-t}$



$e^{j\theta} = \cos\theta + j\sin\theta$

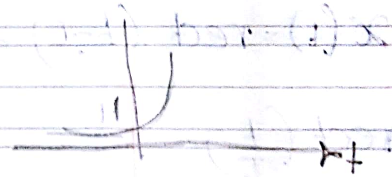
$e^{-j\theta} = \cos\theta - j\sin\theta$

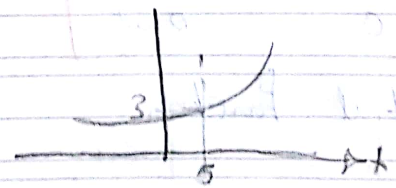
$e^{0}=1\ ,\ e^{\infty}\ ,\ e^{-\infty}=0$

Ex: Sketch the Following Signals

[1] $x(t) = 3e^{t-5}$

$e^{t} \xrightarrow{t \to t-5} e^{t-5}$

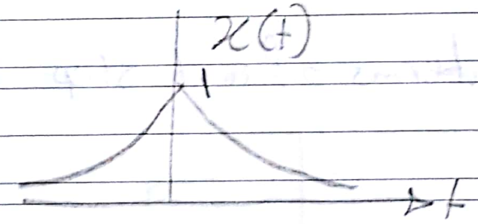




|2| $x(t) = e^{-|t|}$

$$|t| = \begin{cases} t & t \geq 0 \\ -t & t < 0 \end{cases}$$

$$x(t) = \begin{cases} e^{-t} & t \geq 0 \\ e^{t} & t < 0 \end{cases}$$



$$|\square| = \begin{cases} \square & , \square \geq 0 \\ -\square & , \square < 0 \end{cases}$$

$$E_x = \int_{-\infty}^{\infty} x^2(t)\,dt \Rightarrow \int_{-\infty}^{0} [e^{t}]^2 dt + \int_{0}^{\infty} [e^{-t}]^2 dt$$

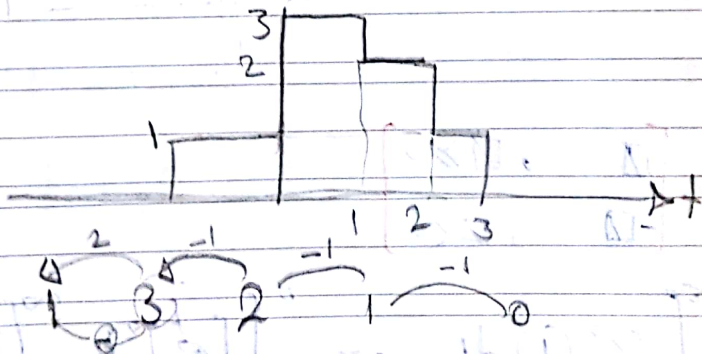
$$= \int_{-\infty}^{0} e^{2t} dt + \int_{0}^{\infty} e^{-2t} dt$$

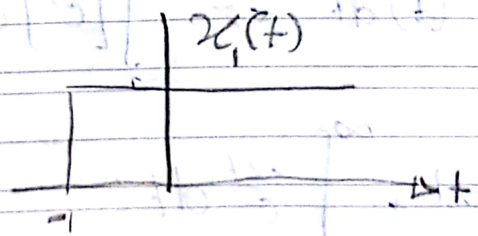
$$= \left[\frac{e^{2t}}{2}\right]_{-\infty}^{0} + \left[\frac{e^{-2t}}{-2}\right]_{0}^{\infty}$$

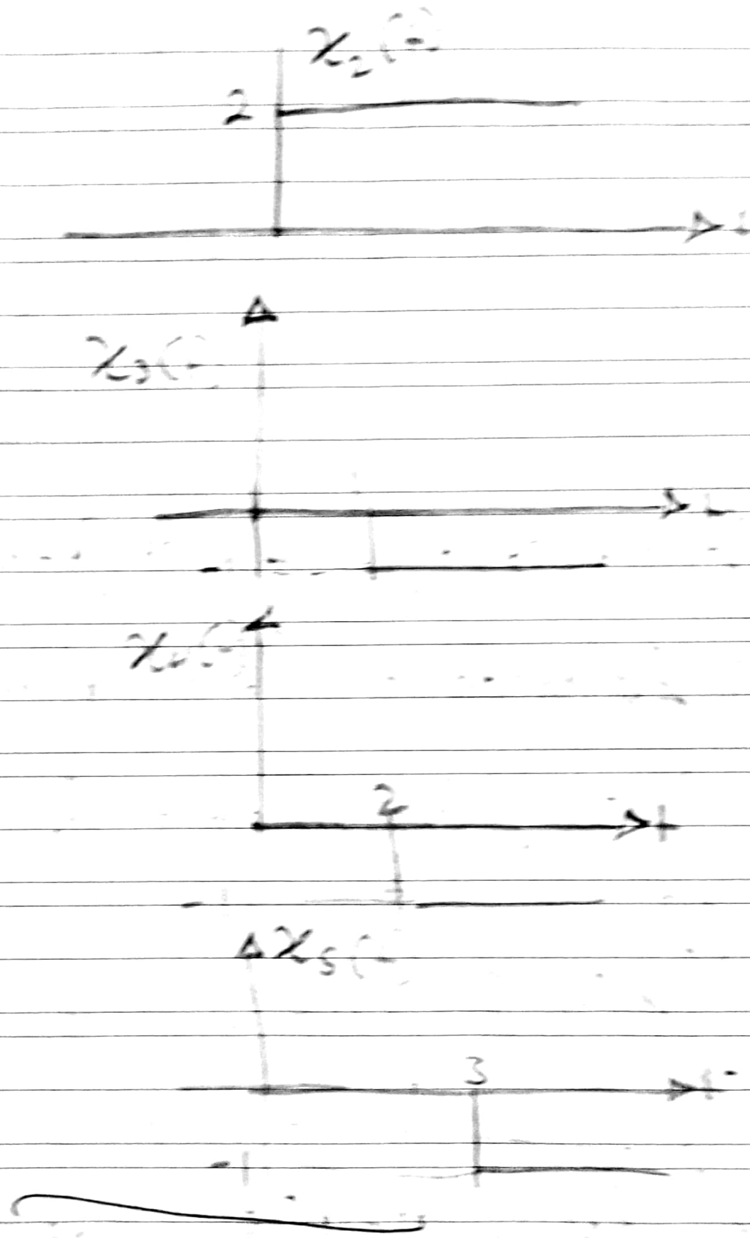
$$= \frac{1}{2}[1-0] - \frac{1}{2}[0-1] = \frac{1}{2} + \frac{1}{2} = 1$$

Ex test I_∞

Q: Express the signal $x(t)$ shown in figure interms of unit step functions



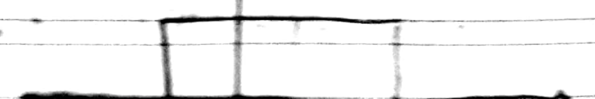




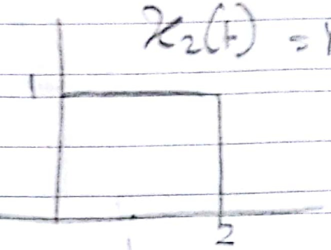
rect - [illegible]

$x_1(t) = rect\left(\frac{t-1}{4}\right)$

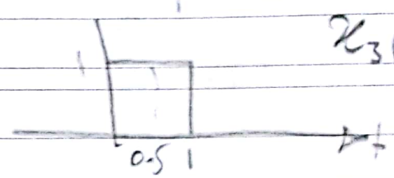
$x_1(t) =$



$x_2(t) = rect\left(\frac{t-1}{2}\right)$



$x_3(t) = \left(\frac{t-0.5}{1}\right)$



$x(t) = rect\left(\frac{t-1}{4}\right) + rect\left(t-0.5\right) + rect\left(\frac{t-1}{2}\right)$

Q] Evaluate the following integral

[1] $\int_{-\infty}^{\infty} e^{-t}\,\delta(\underbrace{2t-2}_{t=1})\,dt = e^{-1}$

$\int_{-\infty}^{\infty} x(t)\,\delta(t-k)\,dt = x(k)$

[2] $\int_{-\infty}^{\infty} \left[t^2 + \cos(\pi t)\right]\delta(\underbrace{t-1}_{t=1})\,dt$

$= (1)^2 + \cos(\pi) = 1 - 1 = 0$

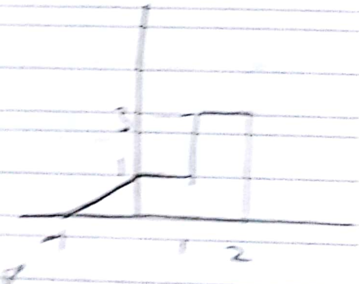
(1) Continues time signal $x(t)$ as shown below sketch and label each of the following signal.

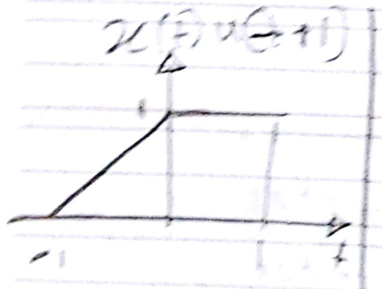
[1] $x(t)\, u(1-t)$

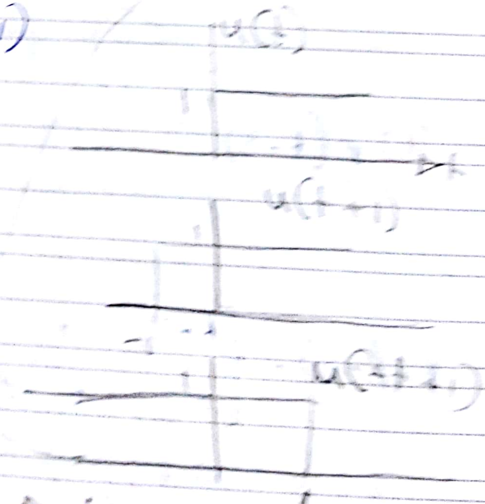
[2] $x(0.5(t-1))$ or $(t-3)$

[3] $x(t-1)\, rect\left(t-\frac{1}{2}\right)$

[1] $u(1-t) = u(-t+1)$



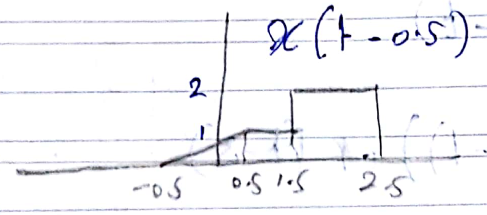


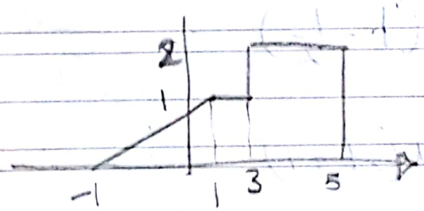


[2] $x(0.5t-0.5)$

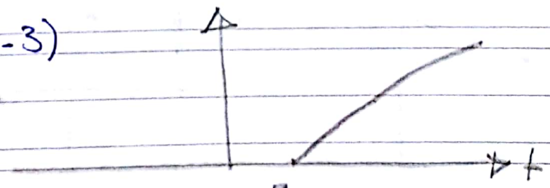
$x(t) \xrightarrow{t \to t-0.5} x(t-0.5)$

$x(0.5t-0.5) \xleftarrow{t \to 0.5t}$

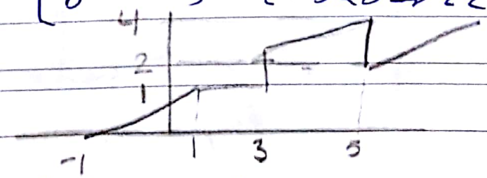




[1] $r(t-3)$



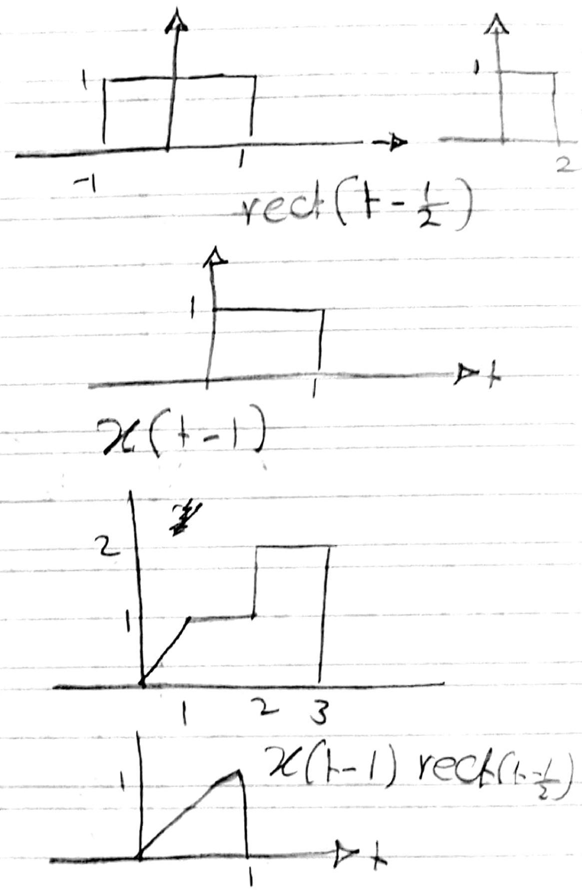
$$r(t-3) = \begin{cases} t-3, & t-3 \geq 0 \to t \geq 3 \\ 0, & t-3 < 0 \to t < 3 \end{cases}$$



$$x(t) = \frac{1}{2}r(t+1) - \frac{1}{2}r(t-1) + u(t-3) + r(t-3) - 2u(t-5)$$

[3] $x(t-1)\,\text{rect}(t-\frac{1}{2})$

$\text{rect}\left(\frac{2t-1}{2}\right)$

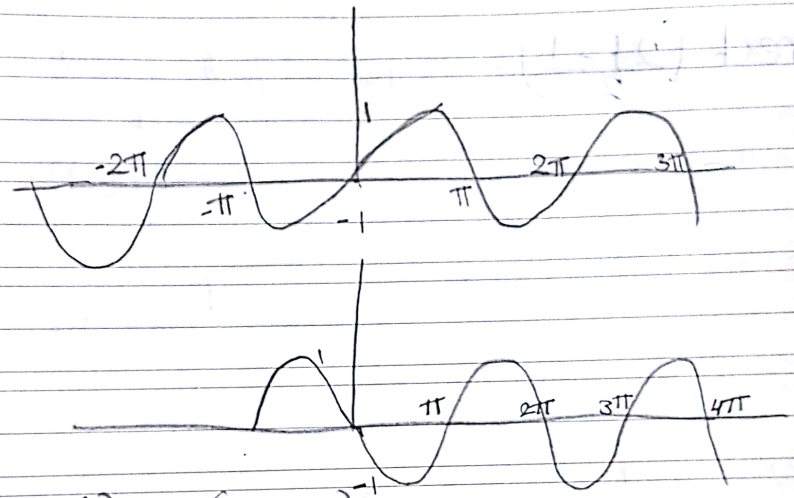


① Conside

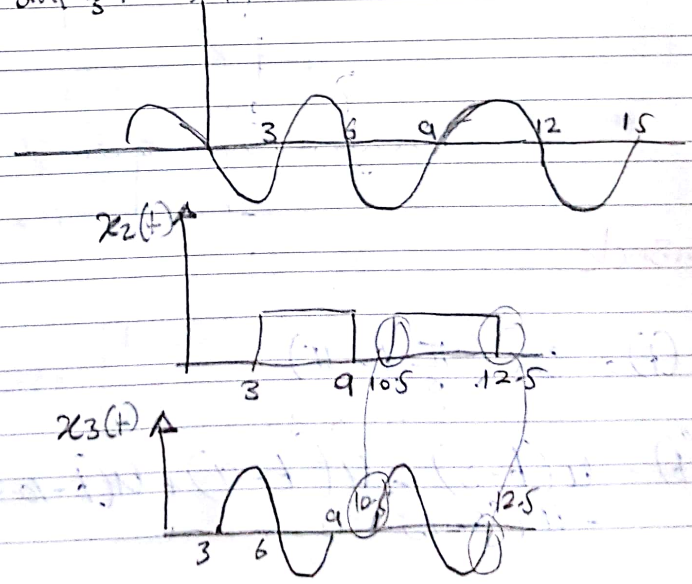
$x_1(t) = \sin\left(\frac{\pi}{8}t - \pi\right)$

$x_2(t) = u(t-3) - u(t-9) + u(t-10.5)$
$- u(t-13.5)$

Sketch $x_3(t) = x_1(t) \cdot x_2(t)$



$x_1(t) = \sin\left(\frac{\pi}{3}t - \pi\right)$



Q] Evaluate the following integral

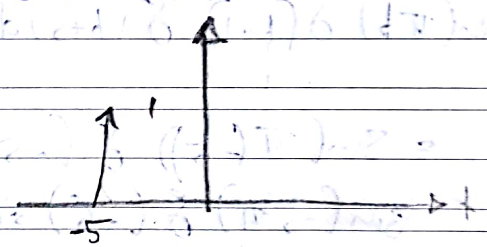
$$\int_{-\infty}^{\infty} [u(t-6)-u(t-10)]\sin\left(\frac{3\pi t}{4}\right)\delta(t-5)\,dt$$

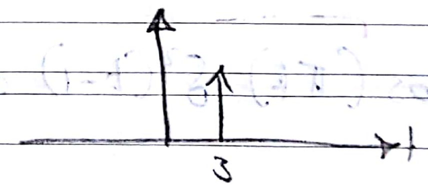
$t=5$

$$= [u(5-6)-u(5-10)]\ \sin\left(\frac{2\pi}{4}5\right)$$

$$= [u(-1)-u(-5)]\ \sin\left[\frac{15}{4}\pi\right]$$

Ex $\displaystyle\int_{-\infty}^{\infty} \delta(t-3)\,\delta(t+5)\,dt$





[illegible] $= 0$

$$\int_{-\infty}^{\infty}$$

$$\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & t \neq 0 \end{cases}$$

$$\delta(0) = 1$$

$$\delta(1) = 0$$

$$\delta(-1) = 0$$

$$\delta(\boxtimes) = \begin{cases} 1 & \boxtimes = 0 \\ 0 & \boxtimes \neq 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \sin(\pi t)\, \delta(t-1)\, \delta(t+5)\, dt$$

$$= \sin(\pi(-5))\, \delta(-5-1)$$

$$\sin(-5\pi)\, \delta(-6) = 0$$

$$\int_{-\infty}^{\infty} \cos(\pi t)\, \delta^2(t-1)\, dt$$

$$\int_{-\infty}^{\infty} \cos(\pi t)\, \delta(t-1)\, \delta(t-1)\, dt$$

$$= \cos(\pi)\, \delta(1-1) = -1$$

$$\int_{3}^{4} \left[\delta(t-1) + \cos(\pi t) + \delta(t+2)\right] \delta(t+2)\, dt$$

$$= 0$$

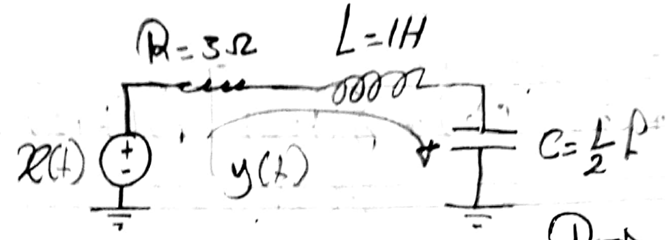
لأن حدود التكامل من $-\infty$ إلى ∞

$$= \delta(-2-1) + \cos(-2\pi) + \delta(-2+2)$$

$$= \delta(-3) + 1 + 1 = 2$$

4-4-2018

تنسيم



$R \Rightarrow V = I R$

$i/P = x(t)$ $\qquad$ $C \Rightarrow V = \frac{1}{C}\int_{-\infty}^{t} i(t)\, dt$

$o/P = y(t)$ $\qquad$ $L \Rightarrow V = L\frac{di}{dt}$

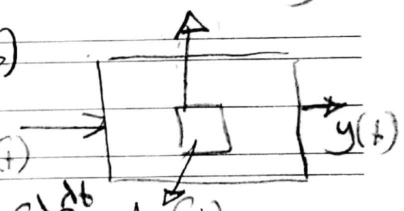
$x(t) = V_R(t) + V_L(t) + V_C(t)$

$x(t) = R y(t) + L\frac{dy}{dt} + \frac{1}{C}\int_{-\infty}^{0} y(t)\, dt$

$x(t) = 3y(t) + \frac{dy}{dt} + 2\int_{-\infty}^{t} y(t)\, dt$

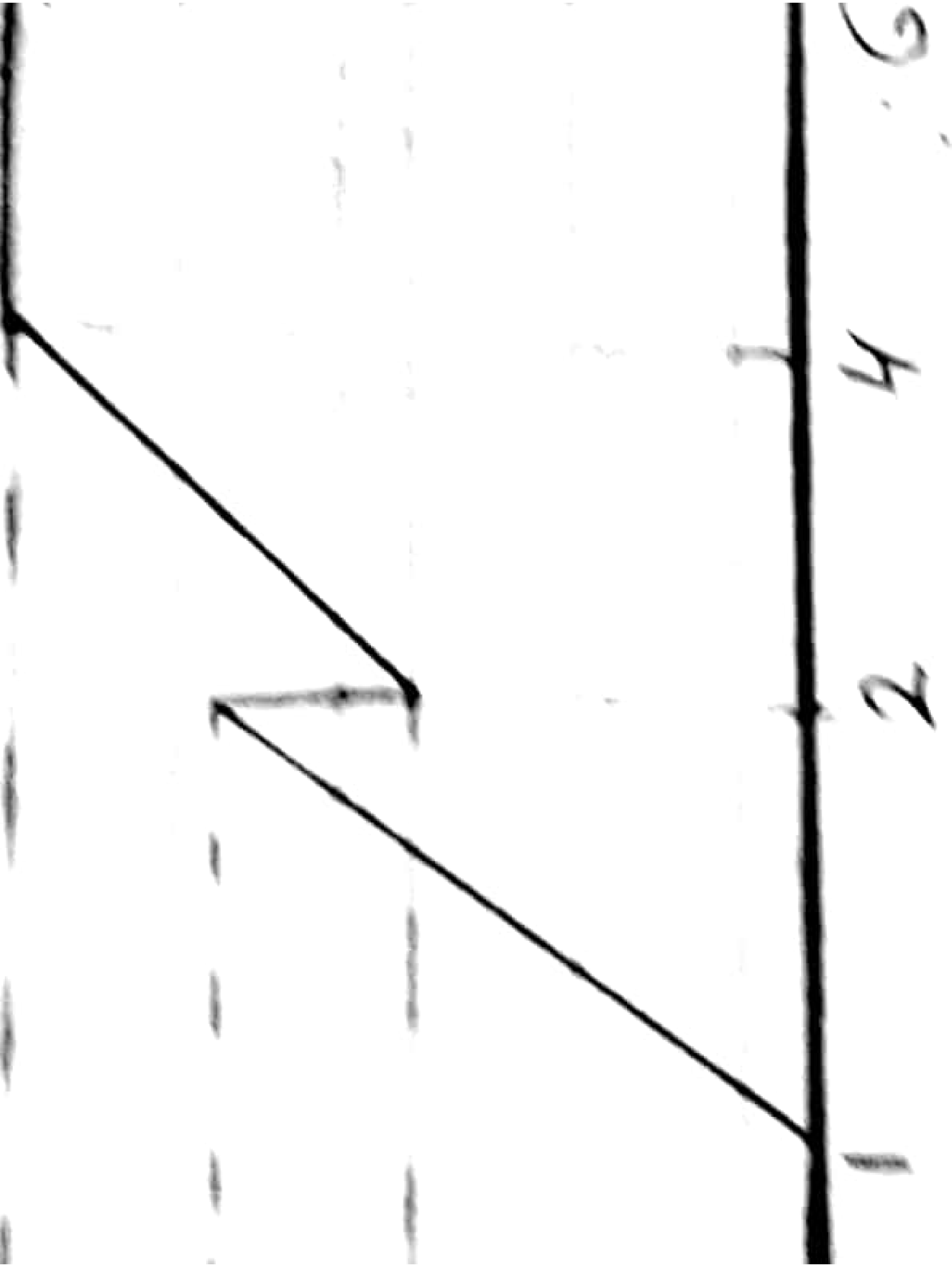
$\Rightarrow \frac{dx}{dt} = 3\frac{dy}{dt} + \frac{d^2y}{dt^2} + 2y(t)$

LTIC system

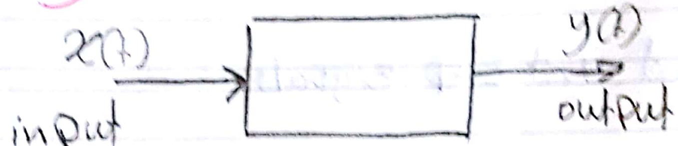


$$\boxed{\frac{dy}{dt} = Dy(t) = \frac{d}{dt}\left(ce^{\lambda t}\right) = c\lambda e^{\lambda t}}$$

$$\frac{d^2y}{dt^2} = D^2y(t) = \frac{d}{dt}\left(c\lambda e^{\lambda t}\right) = c\lambda^2 e^{\lambda t}$$


4
2
1

* System :



→ Classifications of systems :

[1] linear or Nonlinear :
خطي — غير خطي

$x_1(t) \longrightarrow y_1(t)$

$x_2(t) \longrightarrow y_2(t)$

$x_1(t)\, x_2(t) \longrightarrow y_3(t)$

if $y_3(t) = y_1(t) + y_2(t)$

∴ System is linear

else

System is Non linear

[2] Causal or non Causal

سببي : غير سببي

← عندما يكون زمن (output) هو نفسه زمن (input)
او بعد الزمن (input) والنظام يكون سببي Causal

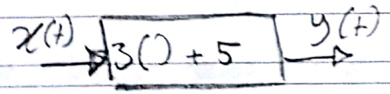
$t_{output} \geq t_{input}$

← والعكس يكون غير سببي (nonCausal)

حتى Causal افرض قيمة لـ t عدد فعلي وعوض فيها وشوف زمن output

Ex

$y(t) = 3x(t-2) + 5$



Delay(2)

$x_1(t) \rightarrow y_1(t) = 3x_1(t-2) + 5$

$x_2(t) \rightarrow y_2(t) = 3x_2(t-2) + 5$

$x_1(t) + x_2(t) \xrightarrow{y_3(t)} 3\left[x_1(t-2) + x_2(t-2)\right] + 5 =$

$3x_1(t-2) + 3x_2(t-2) + 5$

$$y_1(t) + y_2(t) = 3x_1(t-2) + 3x_2(t-2) + 10$$

$$y_3(t) \neq y_1(t) + y_2(t)$$

System is Non linear.

$t = 0 \rightarrow y(0) = 3x(-2) + 5$

$t = 1 \rightarrow y(1) = 3x(-1) + 5$

$t = -1 \rightarrow y(-1) = 3x(-3) + 5$

Causal system.

Ex $\frac{dy(t)}{dt} + 3y(t) = x(t)$

$x_1(t) \rightarrow \frac{dy_1(t)}{dt} + 3y_1(t) = x_1(t)$

$x_2(t) \rightarrow \frac{dy_2(t)}{dt} + 3y_2(t) = x_2(t)$

$x_1(t) + x_2(t) \rightarrow \frac{dy_3(t)}{dt} + 3y_3(t) = x_1(t) + x_2(t)$

$$\frac{d}{dt}\left(y_1(t)+y_2(t)\right)+3\left(y_1(t)+y_2(t)\right)$$

$$=x_1(t)+x_2(t)$$

$$\frac{dy_1(t)}{dt}+\frac{dy_2(t)}{dt}+3y_1(t)+3y_2(t)=x_1(t)+x_2(t)$$

Linear

تمرين تسليم

Ex $y(t)\frac{dy(t)}{dt}+3y(t)=x(t)$

$$x_1(t)\longrightarrow y_1(t)\frac{dy_1(t)}{dt}+3y_1(t)=x_1(t)$$

$$x_2(t)\longrightarrow y_2(t)\frac{dy_2(t)}{dt}+3y_2(t)=x_2(t)$$

$$x_1(t)+x_2(t)\longrightarrow y_3(t)\frac{dy_3(t)}{dt}+3y_3(t)=x_1(t)+x_2(t)$$

$$\left[y_1(t)+y_2(t)\right]\frac{d}{dt}\left[y_1(t)+y_2(t)\right]+3\left[y_1(t)\right.$$

$$\left.+y_2(t)\right]=x_1(t)+x_2(t)$$

$$y_1(t)\frac{d}{dt}\left(y_1(t)+y_2(t)\right)+y_2(t)\frac{d}{dt}\left(y_1(t)y_2(t)\right)$$

$$+3y_1(t)+3y_2(t)=x_1(t)+x_2(t)$$

Non linear

Ex $y(t) = \sin(t)\, x(t-2)$

$x_1(t) \rightarrow y_1(t) = \sin(t)\, x_1(t-2)$

$x_2(t) \rightarrow y_2(t) = \sin(t)\, x_2(t-2)$

$x_1(t) + x_2(t) \rightarrow y_3(t) = \sin(t)\left[x_1(t-2) + x_2(t-2)\right]$ linear & Causal

Ex $y(t) = \sin(t) + x(t-2)$

$x_1(t) \longrightarrow y_1(t) = \sin(t) + x_1(t-2)$

$x_2(t) \longrightarrow y_2(t) = \sin(t) + x_2(t-2)$

$x_1(t) + x_2(t) \longrightarrow y_3(t) = \sin(t) + x_1(t-2) + x_2(t-2)$

Nonlinear

Ex: $y(t) = 3e^{x(t)}$

$x_1(t) \rightarrow y_1(t) = 3e^{x_1(t)}$

$x_2(t) \longrightarrow y_2(t) = 3e^{x_2(t)}$

$x_1(t) + x_2(t) \longrightarrow y_3(t) = 3e^{x_1(t) + x_2(t)}$

$= 3e^{x_1(t)\, x_2(t)}$

Nonlinear

Ex: $\int_{-\infty}^{\infty} y(t-3)\,dt = x(t+5)$

$x_1(t) \rightarrow \int_{-\infty}^{\infty} y_1(t-3)\,dt = x_1(t+5)$

$x_2(t) \rightarrow \int_{-\infty}^{\infty} y_2(t-3)\,dt = x_2(t+5)$

$x_1(t) + x_2(t) \rightarrow \int_{-\infty}^{\infty} y_3(t-3)\,dt = x_2(t+5) + x_2(t+5)$

$\int_{-\infty}^{\infty} \left[y_1(t-3) + y_2(t-3) \right] dt = x_1(t+5) + x_2(t+5)$

$\int_{-\infty}^{\infty} y_1(t-3)\,dt + \int_{-\infty}^{\infty} y_2(t-3)\,dt = x_1(t+5) + x_2(t+3)$

linear

كل Memoryless هو Causal والعكس صحيح

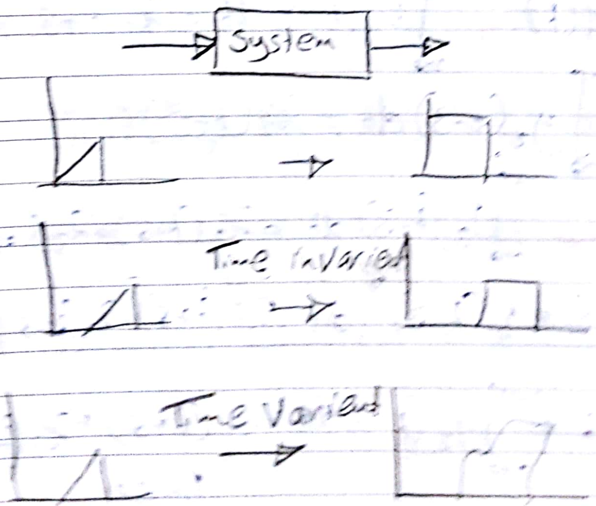
[3] Memory or Memoryless

- اذا كان زمن (output) هو نفسه زمن (input) فإن النظام يكون بدون ذاكرة (Memoryless)
- عدا ذلك النظام فيه ذاكرة (Memory)

Every Memoryless systems is Causal system.

[4] Time Varient & Time invarient

متغير مع الزمن — غير متغير مع الزمن



5 Invertable & Noninvertable

قابل للانعكاس   غير قابل للانعكاس

Ex: $y(t) = x(t) + 5$

$x(t) = y(t) - 5$

$y(t) = (x(t)) - 5$

$$x(t) \longrightarrow \boxed{(\ )+5} \xrightarrow{y(t)} \boxed{(\ )-5} \longrightarrow x(t)$$

Ex: $y(t) = x^2(t)$

$x(t) = \pm\sqrt{y(t)}$

$$x(t) \longrightarrow \boxed{(\ )^2} \xrightarrow{y(t)} \boxed{\pm\sqrt{\ \ }} \longrightarrow x(t)$$

Non Invertable

Ex:

$y(t) = 5x^3(t)$

$x(t) = \cancel{\tfrac{1}{5}}\sqrt[3]{\frac{y(t)}{5}}$

$$x(t) \longrightarrow \boxed{5(\ )^3} \longrightarrow \boxed{\sqrt[3]{\tfrac{\ }{5}}} \longrightarrow x(t)$$

$y(t) = \cancel{\tfrac{1}{5}}\sqrt[3]{\frac{x(t)}{5}}$

Invertable

Ex: $y(t) = 15 \ln(x(t))$

$$\frac{y(t)}{15} = \ln(x(t))$$

$$e^{\frac{x(t)}{15}} = x(t)$$

$$y(t) = e^{\frac{x(t)}{15}}$$

$$x(t) \to \boxed{15\ln(\,)} \xrightarrow{y(t)} \boxed{e^{(\,)}} \to x(t)$$

15] Stable or Non stable
مستقر — غير مستقر
مش معروف لو ثابت stable

BIBO ≡ Bounded Input Bounded output

ZIZO ≡ Zero Input Zero output

Ex: $y(t) = x^2(t)$

$x(t) = B < \infty$

$y(t) = B^2$ stable

Ex: $y(t) = t\,x^2(t)$

$x(t) = B < \infty$

$y(t) = B^2 t$ when $t = \infty \rightarrow y(t) = \infty$

Nonstable

Ex: Check linearity, stability for the following system:

$y(t) = \frac{x(t)}{t+1}$

$x_1(t) \rightarrow y_1(t) = \frac{x_1(t)}{t+1}$

$x_2(t) \rightarrow y_2(t)\ \frac{x_2(t)}{t+1}$

$x_1(t) + x_2(t) \to y_3(t) = \dfrac{x_1(t) + x_2(t)}{t+1}$

$= \dfrac{x_1(t)}{t+1} + \dfrac{x_2(t)}{t+1}$

linear

$t = 0 \to y(0) = \dfrac{x(0)}{0+1} = x(0)$

$t = 1 \to y(1) = \dfrac{x(1)}{1+1} = \dfrac{x(1)}{2}$

$t = -1 \to y(-1) = \dfrac{x(-1)}{-1+1} = \dfrac{x(-1)}{0} = \infty$

Causal + Memoryless + Nonstable

$y(t-k) = \dfrac{x(t-k)}{t-k+1}$

$x(t-k) \to y(t) = \dfrac{x(t-k)}{t+1}$ Tim Varient

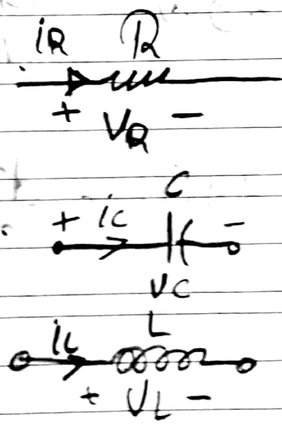
$x(t) \to \boxed{\dfrac{(\ )}{t+1}} \to y(t)$

What is the inverse system if any?

$$y(t) = \frac{x(t)}{t+1}$$

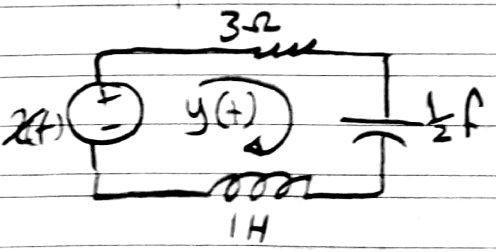
$$x(t) = (t+1)\,y(t)$$

$$y(t) = (t+1)\,x(t)$$



i_R, R, $+\ V_R\ -$	$i_R = \frac{V_R}{R}$	$V_R = R\,i_R$
C, $+\ i_C$, V_C	$i_C = C\frac{dV_C}{dt}$	$V_C = \frac{1}{C}\int i_C\,dt$
L, i_L, $+\ V_L\ -$	$i_L = \frac{1}{L}\int V_L\,dt$	$V_L = L\frac{di_L}{dt}$

EX



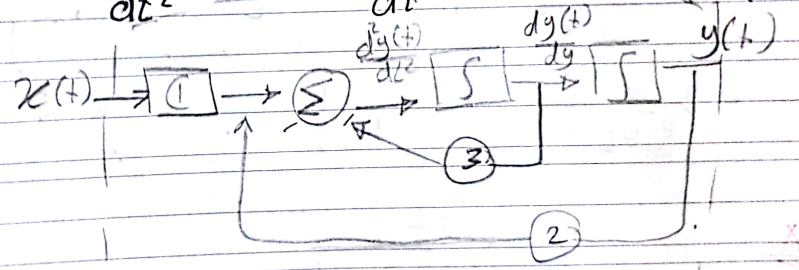
$$x(t) = V_R + V_L + V_C$$

$$x(t) = 3y(t) + (1)\frac{dy(t)}{dt} + \frac{1}{\frac{1}{2}}\int y(t)\,dt$$

$$\frac{dy(t)}{dt} + 3y(t) + 2\int y(t)\,dt = x(t)$$

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = \frac{dx(t)}{dt}$$

$$\frac{d^2y(t)}{dt^2} = -3\frac{dy(t)}{dt} - 2y(t) + \frac{dx(t)}{dt}$$



$m^2 + 3m + 2 = 0$

$(m+2)(m+1) = 0 \qquad m_1 = -2,\ m_2 = -1$

$y(t) = c_1 e^{-2t} + c_2 e^{-t}$

Example: sketch the following signals:

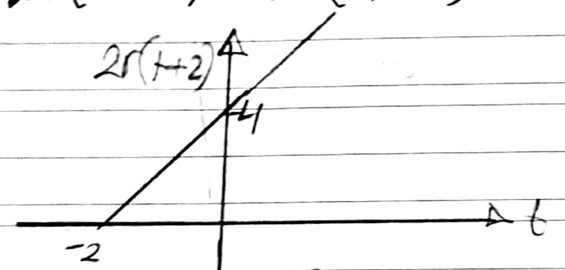
ا.مكرم غني

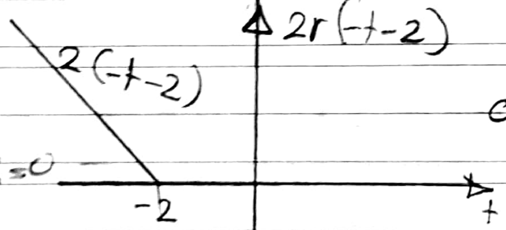
5-4-2018

[1] $x_1(t) = 2r(t+2) + 2r(-t-2) - 6 + u(t)$

$2r(-t-2)$

$t = -2$





$2(-t-2) - 6 = 0$

$\frac{2(-t-2)}{2} = \frac{6}{2}$

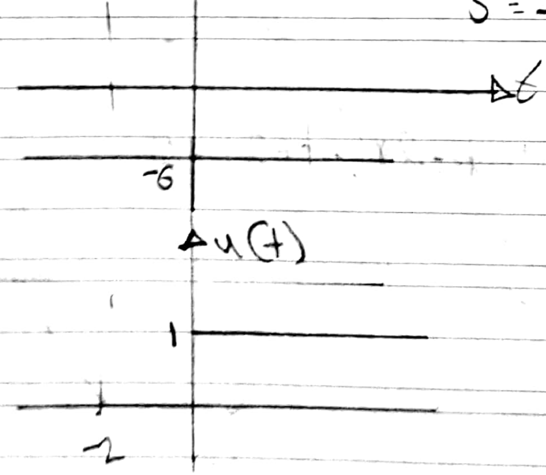
$-t-2 = 3$

$t = -5$

$0 = -6 + 2(-t-2)u(-t-2)$

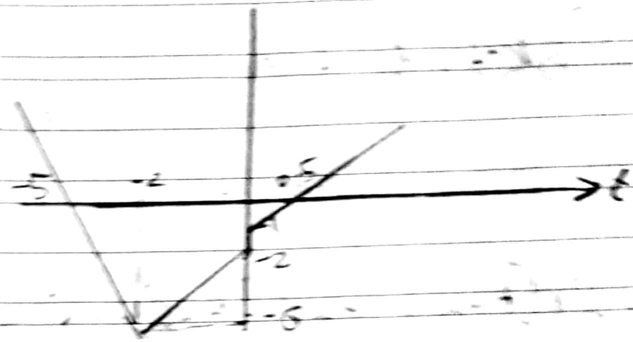
$6 = 2(-t-2)u(-t-2)$

$3 = -t-2$



$2r(-)$ [illegible] -2 $\quad$ $2r(1-t)$ [illegible] -2

$-2r(-)$ $\quad -2$ $\quad$ $-2r(1-t)$ [illegible] $+2$



$-2 \leq t \leq 0$

$2r(t+2)-6 = 2(t+2)\,u(t+2)-6$

at $t=0 \Rightarrow 2(0+2)\,u(2)-6 =$

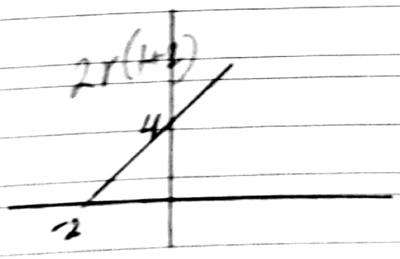
$4-6 = -2$

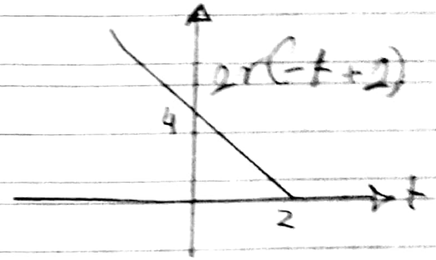
at $t=-2 \Rightarrow 2(-2+2)\,u(0)-6 = -6$

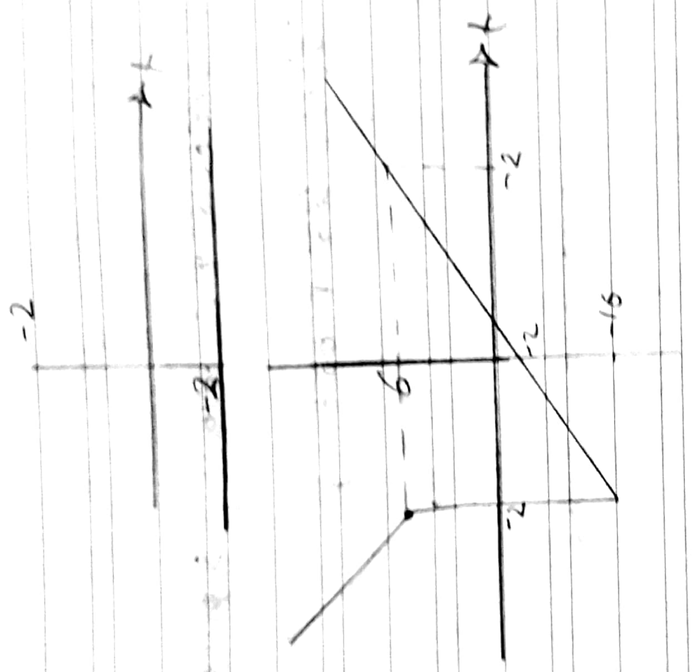
$t \geq 0$

$2r(t+2)-5 = 2(t+2)\,u(t+2)-5 = 0$

$\boxed{2}$ $2r(t+2)-2r(-t+2)-2$





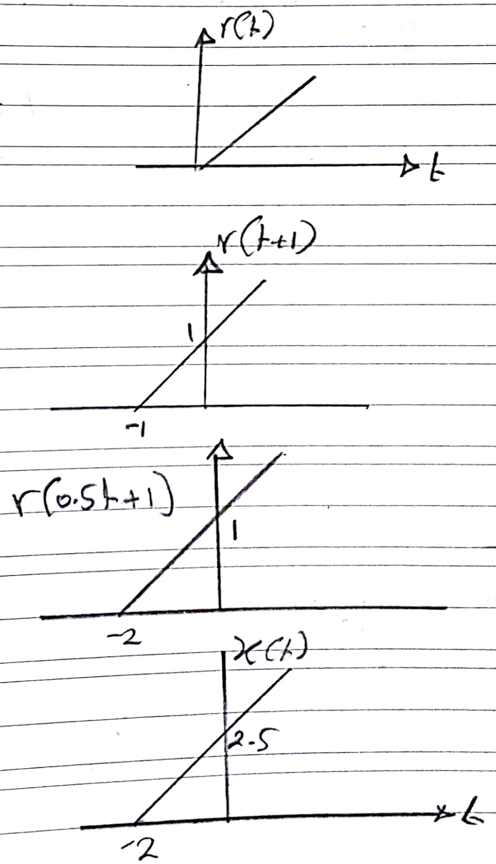


المزوغي

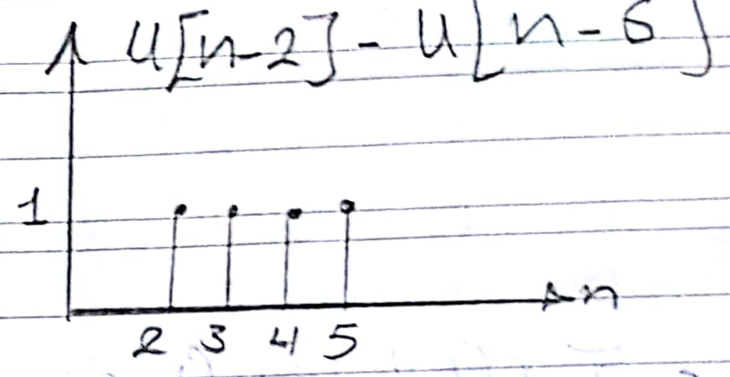
7.4.2018

Ex: Sketch the following signals:

1] $x(t) = 2.5\, r(0.5t+1)$



2] $x[n] = (n-2)\,[u(n-2) - u(n-6)]$



$n = 2 \rightarrow x[n] = 0$

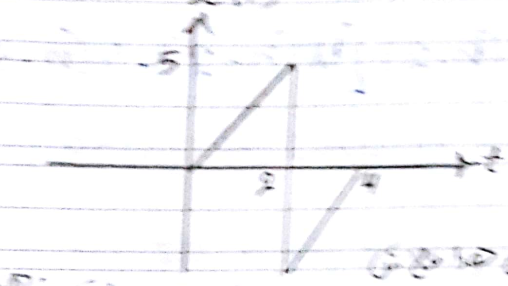
$n = 3 \rightarrow x[n] = 1$

$n = 4 \rightarrow x[n] = 2$

$n = 5 \rightarrow x[n] = 3$

Ex Express the signals in terms of unit step and ramp function

2]



$x(t) = \frac{5}{2} r(t) - 10\, u(t-2) - \frac{5}{2} r(t-4)$

Ex Find the Pav & E for the following signals

1] $x_1(t) = 2r(t)$

neither energy nor power

$E_{x_1} = \int_{-\infty}^{\infty} x_1^2(t)\, dt$

$2\int_0^{\infty} t^2\, dt = \frac{2}{3}\left[t^3\right]_0^{\infty} = \infty$

$P_x = \lim_{T\to\infty} \frac{1}{T} \int_0^{T} x_1^2(t)\, dt$

$$= \lim_{T \to \infty} \frac{2}{T} \int_0^T t^2 \, dt$$

$$= \lim_{T \to \infty} \frac{2}{T} \left[\frac{t^3}{3} \right]_0^T$$

$$= \lim_{T \to \infty} \frac{2}{T} \quad \frac{T^3}{3} = \infty$$

[2] $x_2(t) = 4 \cos\left(\frac{2\pi}{10} t \right)$

$$\frac{T_1}{T_2} = \frac{3}{2} \qquad \text{Periodic}$$

لو T_1 و T_2 اعداد صحيحة كان غير حقيقي

non Periodic

$$T = 3T_2 = 2T_1$$

$$T = 3 \times 2 = 2 \times 3 = 6$$

$$\frac{T_1}{T_2} = \frac{N_1}{N_2}$$

$$T = T_1 N_2 = T_2 N_1$$

$$x(t) = \cos(2\pi t) - \sin(3t)$$

$W_1 = \cancel{2\pi} = \frac{2\pi}{T}$	$W_2 = 3 = \frac{2\pi}{T}$
$T_1 = 1$	$T_2 = \frac{2\pi}{3}$

$$\frac{T_1}{T_2} = \frac{1}{\frac{2\pi}{3}} = \frac{3}{2\pi}$$

non periodic

Ex : Express $z(t) = x(t) + y(t)$

as a single sinusoid :

$x(t) = 3\cos(14\pi t - 0.2)$

$y(t) = 2\cos(14\pi t + 0.1)$

$\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y)$

$3\cos(14\pi t - 0.2) = 3\cos(14\pi t)\cos(0.2) +$

$3\sin(14\pi t)\sin(0.2)$

$x(t) = 2.94\cos(14\pi t) + 0.596\sin(14\pi t)$

$y(t) = 2\cos(14\pi t)\cos(0.1) - 2\sin(14\pi t)\sin(0.1)$

$y(t) = 2\cos(14\pi t) - 0.4\sin(14\pi t)$

$$= 4.94\cos(14\pi t) - 0.196\sin(14\pi t)$$

$$A\cos(\theta) + B\sin(\theta) = C\cos(\theta + \alpha)$$

$$C = \sqrt{A^2+B^2}, \quad \alpha = -\tan^{-1}\left(\frac{B}{A}\right)$$

$$C = \sqrt{4.94^2 + 0.196^2} = 4.96$$

$$\alpha = -\tan^{-1}\left(\frac{-0.196}{4.94}\right) = 2.27^{\circ}$$

$$x(t) = 4.96\cos(14\pi t + 2.27^{\circ})$$

Ex: Express $x(t)$ in terms of unit step

II $$x(t) = \begin{cases} e^{-|t|}, & t < -2 \\ 0, & t > -2 \end{cases}$$

$$u(-t-2)$$
$$t = -2$$

$$x(t) = e^{t}\, u(-t-2)$$

$$|t| = \begin{bmatrix} t & , t \geq 0 \\ -t & , t \leq 0 \end{bmatrix}$$

$|t| = t$ for $t \leq 0$

$|t| = -t$ for $t < 0$

2]

$$x(t) = \begin{vmatrix} 2 & -1 < t < 1 \quad |t| < 1 \\ (3-t) & 1 < t < 2 \\ 0 & \text{o.w} \end{vmatrix}$$

$$x(t) = 2\left[u(t+1) - u(t-1)\right] + (3-t)$$

$$\left[u(t-1) - u(t-2)\right]$$

3]
$$\int_{-1}^{1} (3t^2+1)\, \delta(t)\, dt = 1$$

$$\int_{-\infty}^{\infty} \left[t^2 + \cos(\pi t)\right] \delta(t-1)\, dt$$

$$= (1)^2 + \cos(\pi) = 0$$

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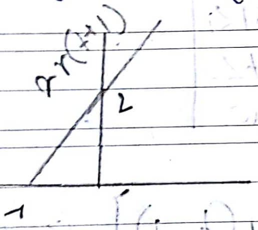
$$\int_{-\infty}^{\infty} \sin(t-1)\,\delta(2t-4)\,dt \qquad t=2$$

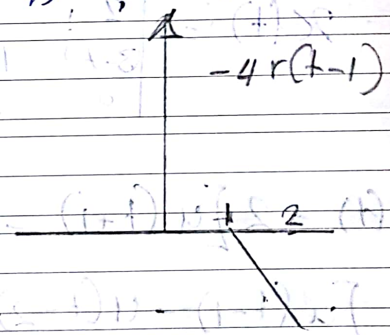
$$= \sin(1)$$

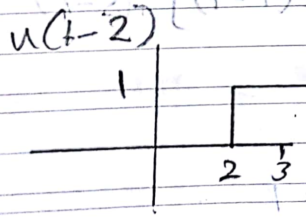
Ex

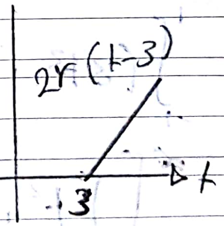
Sketch

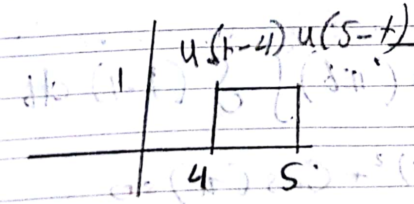
$$x(t) = 2r(t+1) - 4r(t-1) + u(t-2) + 2r(t-3) - u(t-4)\,u(5-t)$$











$-1 \leq t \leq 1$

$x(t) = 2r(t+1) = 2(t+1)u(t+1)$

$t = 1 \Rightarrow x(t) = 4$

$1 \leq t \leq 2$

$x(t) = 2r(t+1) - 4r(t-1)$

$= 2(t+1)u(t+1) - 4(t-1)u(t-1)$

$t = 1 \rightarrow x(t) = 4$

$t = 2 \rightarrow x(t) = 6 - 4 = 2$

$2 \leq t \leq 3$

$x(t) = 2r(t+1) - 4r(t-1) + u(t-2)$

$= 2(t+1)u(t+1) - 4(t-1)$
$u(t-1) + u(t-2)$

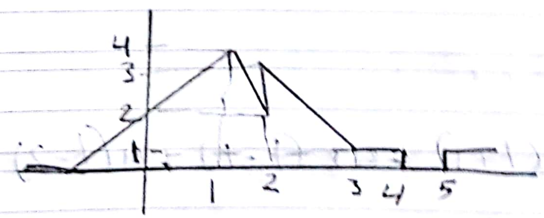
$t=2 \rightarrow x(t)=3$

$t=3 \rightarrow x(t)=1$

$3 \leq t \leq 4$

$x(t) = 2r(t+1) - 4r(t-1) + u(t-2)$
$+ 2r(t-3)$

$t=3 \rightarrow x(t)=1$
$t=4 \rightarrow x(t)=1$



Ex systems

$$y(t) = \int_{t-1}^{t} x(\tau)\, d\tau$$

$$x_1(t) \rightarrow y_1(t) = \int_{t-1}^{t} x_1(\tau)\, d\tau$$

$$x_2(t) \rightarrow y_2(t) = \int_{t-1}^{t} x_2(\tau)\, d\tau$$

$$x_1(t) + x_2(t) \rightarrow y_3(t) = \int_{t-1}^{t} [x_1(\tau) + x_2(\tau)]$$

linear

note that $y(t)$ is depends on x at t & $t-1$ Causal

Memory

$$x(t) = B < \infty$$

$$y(t) = \int_{t-1}^{t} B\, dt = B(t - t + 1) = B < \infty$$

stable

Invertable

$$\frac{dy(t)}{dt} = \frac{d}{dt}\int_{t-1}^{t} x(\tau)\,d\tau$$

$$\frac{dy(t)}{dt} = x(t) - x(t-1)$$

Time invarient

Ex $y(t) = 3x(3t+3)$

linear

$x_1(t) \rightarrow y_1(t) = 3x_1(3t+3)$

$x_2(t) \rightarrow y_2(t) = 3x_2(3t+3)$

$x_1(t) + x_2(t) \rightarrow y_3(t) = 3[x_1(3t+3) + x_2(3t+3)]$

Non Causal

Memory

Time Invarient

$y(t-k) = 3x(3(t-k)+3)$

$x(t-k) \rightarrow \hat{y}(t) = 3x(3(t-k)+3)$

Time Inverient:

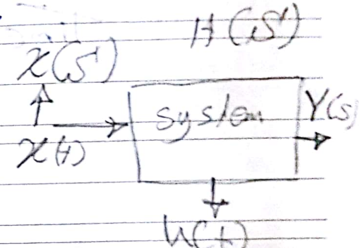
تيسير

The Convolution Integral :

2018 / 4 / 8

$$y(t) = x(t) * h(t)$$

$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

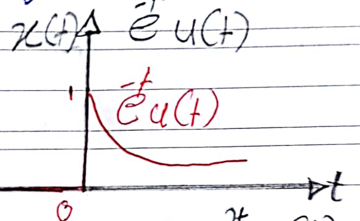


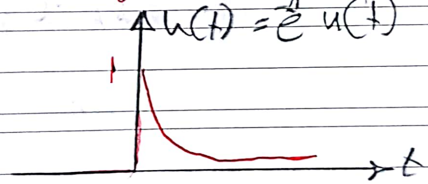
$Y(s) = H(s)\, X(s)$

$x(t) = e^{-5t}$

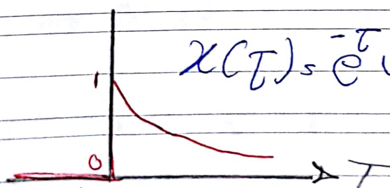
$X(s) = \dfrac{1}{S+5}$

$H(s) = S$





$x(\tau) = e^{-\tau}u(\tau)$

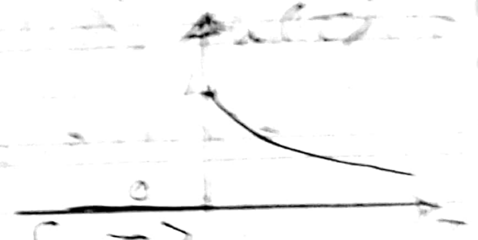


$$y(t) = \int_{0}^{t} \left(e^{-\tau}u(\tau)\right)\left(e^{-2(t-\tau)}u(t-\tau)\right) d\tau$$

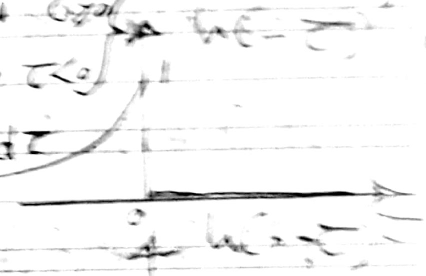
$h(t-\tau) = e^{-2(t-\tau)}\, u(t-\tau)$

$h(t)$

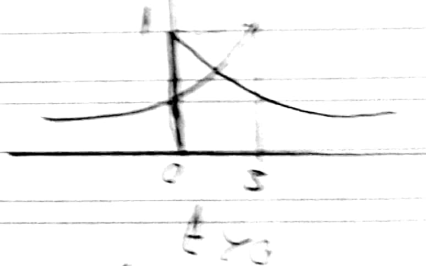
$t \rightarrow (t-\tau)$



$u(\tau) = \begin{cases} 1 & \tau \geq 0 \\ 0 & \tau < 0 \end{cases}$



$y(t) = \int_0^t (e^{-\tau} u(\tau))(e^{-2(t-\tau)} u(t-\tau))\, d\tau$



$\bullet\ t<0 \Rightarrow y(t) = 0$

$t \geq 0 \quad y(t) = \int_0^t e^{-\tau} e^{-2(t-\tau)}\, d\tau$

$t>0$

$= e^{-2t} \int_0^t e^{\tau}\, d\tau = e^{-2t} (e^{\tau}) \Big|_0^t$

$y(t) = (e^{-t} - e^{-2t}) \qquad t>0$

$y(t) = 0 \qquad t<0$

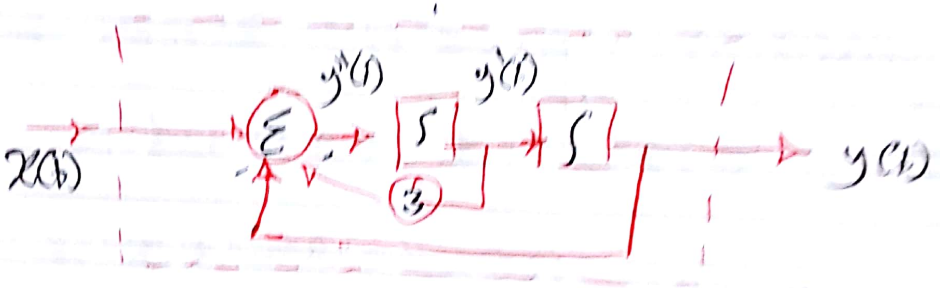
$y(t) = (e^{-t} - e^{-2t})\, u(t)$

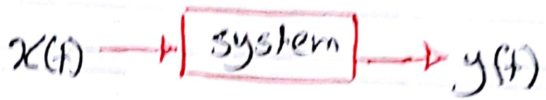
24-4-2018 الأربعاء

⟹ Block Digram representing systems :-

Ex : $y''(t) + 3y'(t) + y(t) = x(t)$

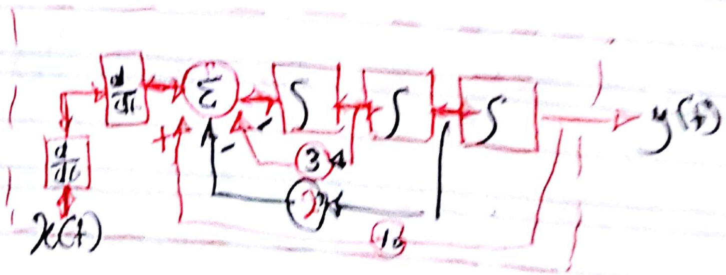
$y''(t) = x(t) - 3y'(t) - y(t)$





Ex : $y'''(t) + 3y''(t) + 2y'(t) - 10y(t) = x''(t)$

$y'''(t) = x''(t) - 3y''(t) - 2y'(t) + 10y(t)$



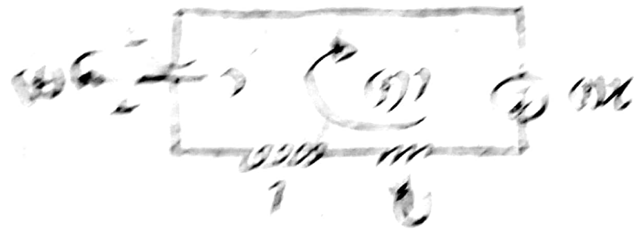
$x(t) \rightarrow$ system $\rightarrow y(t)$

$$v(t) = v_c = \frac{1}{C}\int i(t)\,dt$$

$$x(t) = \underbrace{R\,i(t)}_{v_R} + \underbrace{L\frac{di(t)}{dt}}_{v_L} + \underbrace{\frac{1}{C}\int i(t)\,dt}_{v_C}$$

L	$v_L = L\frac{di}{dt}$, $i_L = \frac{1}{L}\int v_L\,dt$
C	$v_c = \frac{1}{C}\int i\,dt$, $i_c = C\frac{dv_c}{dt}$
R	$v_R = R\,i$, $i = \frac{v_R}{R}$

$$x(t) = v_R + v_L + v_C$$



II

System : I

Q. Find the equation representing the system

$$i(t) = C\,\frac{dy}{dt}$$

$$x(t) = RC\,\frac{dy(t)}{dt} + LC\,\frac{d\dot{y}(t)}{dt} + y(t)$$

$$LCy'' + RCy' + y = x(t)$$

$$y''(t) + \frac{R}{L}y'(t) + \frac{1}{LC}y(t) = \frac{1}{LC}x(t)$$

if $L = 1H$, $C = \frac{1}{2}F$, $R = 3\Omega$

$$\boxed{y''(t) + 3y'(t) + 2y(t) = 2x(t)}$$

Ex: if $x(t) = e^{-5(t)}u(t)$

Find the total Response if $y(0^-) = 0$, $y'(0^-) = 0$

Total Response of Any system = Zero input Response $y_n(t)$ + Zero state response $y_p(t)$

$$y(t) = y_n(t) + y_p(t)$$

$\Rightarrow$ **To find Zero input Response :-**

$$y''(t) + 3y'(t) + 2y(t) = 0$$

$$m^2 + 3m + 2 = 0$$

$$(m+2)(m+1) = 0$$

$$m_1 = -2 \,,\quad m_2 = -1$$

قانون

$$y_n(t) = C_1 e^{m_1 t} + C_2 e^{m_2 t} + C_3 e^{m_3 t} + \dots$$

$$y_n(t) = C_1 e^{-2t} + C_2 e^{-t} .$$

$$y''(t) + 3y'(t) + 2y(t) = 2 e^{-5t} u(t)$$

To find Zero state responce : $y_p(t)$

if $x(t) = K_1 t^3 + K_2 t^2 + \dots$

$$y_p(t) = B_0 + B_1 t + B_2 t^2 + B_3 t^3$$

if $x(t) = K e^{at}$

$y_p(t) = \beta e^{at}$

if $x(t) = K \sin(\omega t)$

$y_p(t) = \beta_1 \sin(\omega t) + \beta_2 \cos(\omega t)$

Ex

$x(t) = t^2 + 5e^{-3t}$

$y_p(t) = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 e^{-3t}$

$x(t) = 2 e^{-5t} u(t)$ المثال الي قبل جزء

$y_p(t) = \beta e^{-5t}$

$y_p'(t) = \beta(-5) e^{-5t} = -5\beta e^{-5t}$

$y_p''(t) = 25\beta e^{-5t}$

$25\beta e^{-5t} - 15\beta e^{-5t} + 2\beta e^{-5t} = 2 e^{-5t}$

$[12\beta]e^{-5t} = 2e^{-5t}$

$\beta = \frac{2}{12} = \frac{1}{6}$ $\quad \therefore y_p(t) = \frac{1}{6}e^{-5(t)}$

$\because y(t) = y_n(t) + y_p(t)$

$y(t) = C_1e^{-2t} + C_2e^{-t} + \frac{1}{6}e^{-5t}$

$y(0) = 0 \quad , \quad y'(0) = 0$

$\uparrow t \quad \uparrow y$

$0 = C_1 + C_2 + \frac{1}{6}$ $\longrightarrow$ $\boxed{C_1 + C_2 = -\frac{1}{6}}$

$y'(t) = -2C_1e^{-2t} - C_2e^{-t} - \frac{5}{6}e^{-5t}$

$0 = -2C_1 - C_2 - \frac{5}{6} \longrightarrow 2C_1 + C_2 = -\frac{5}{6}$

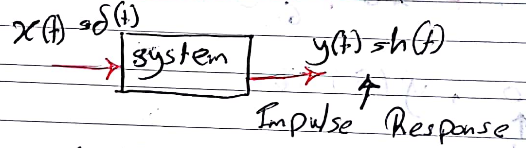
$C_1 = -\frac{5}{6} + \frac{1}{6} = \frac{-4}{6}$

$C_2 = -\frac{1}{6} + \frac{4}{6} = \frac{3}{6} = \frac{1}{2}$

$y(t) = \left[-\frac{4}{6}e^{-2t} + \frac{1}{2}e^{-t} + \frac{1}{6}e^{-5t}\right]u(t)$

⇒ Unit Impulse Response :-
$h(t)$

⇒ Any Signal Can be represented as sum of impulses.



$\therefore h(t) = y(t)$ when $x(t) = \delta(t)$.

To find the response of the system to Any Signal $x(t)$

$y(t) = x(t) * h(t) = h(t) * x(t)$

Convolution

$$x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau)\, x_2(t - \tau)\, d\tau$$

Constant

variable

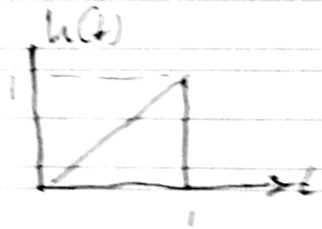
$$x_2(t) * x_1(t) = \int_{-\infty}^{\infty} x_2(\tau)\, x_1(t-\tau)\, d\tau$$

$$\therefore y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$\therefore y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

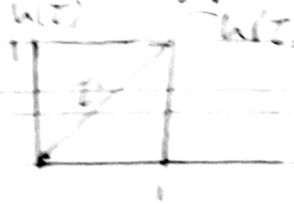
Ex: Find the output of the following System

given



$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

$h(\tau) = \tau$



$$x(t) \rightarrow x(\tau) \xrightarrow{\tau \to \tau+1} x(\tau+1)$$

$x(-\tau+1)$

$x(t-\tau) \quad \tau \to \tau -$

or

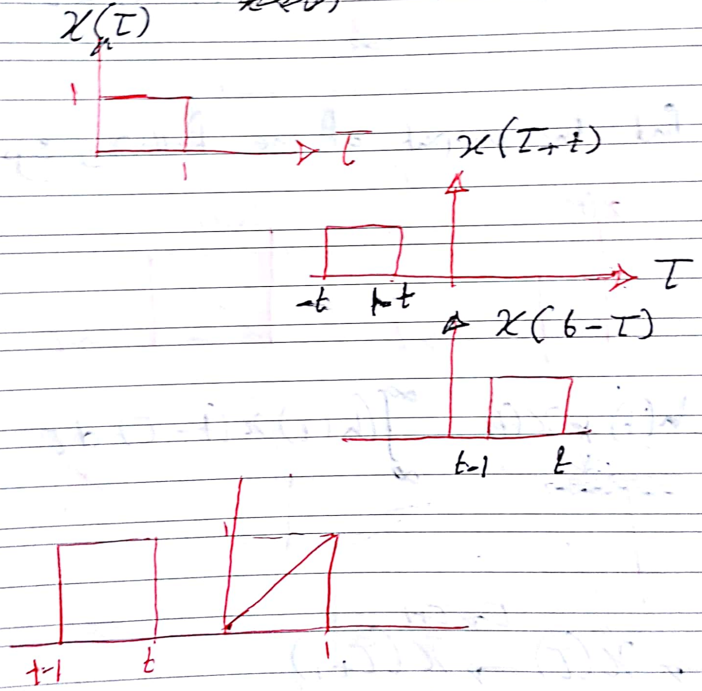
$$x(t) \to x(\tau) \xrightarrow{\tau \to -\tau} x(-\tau)$$

$\tau \to \tau - t$

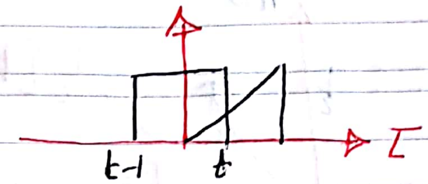
$x(-(\tau - t))$

$x(t-\tau)$

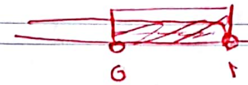
$\cancel{x(\tau)}$



Case (1): $t > 0$

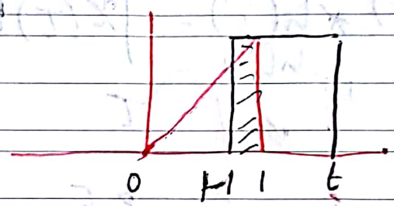


$0 < t < 1$ & $t-1 < 0 \rightarrow t < 1$



$y(t) = \int_0^t (1)\, \tau \, d\tau = \frac{t^2}{2}$

Case(3) :-



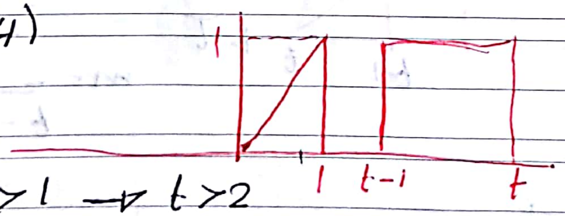
$0 < t-1 < 1$ & ~~t<1~~ $\rightarrow$ $t > 1$

حاصل ضرب العمود الرأسي

$y(t) \int_{t-1}^{1} (1)(\tau)\, d\tau = \left[\frac{\tau^2}{2} \right]_{t-1}^{1}$

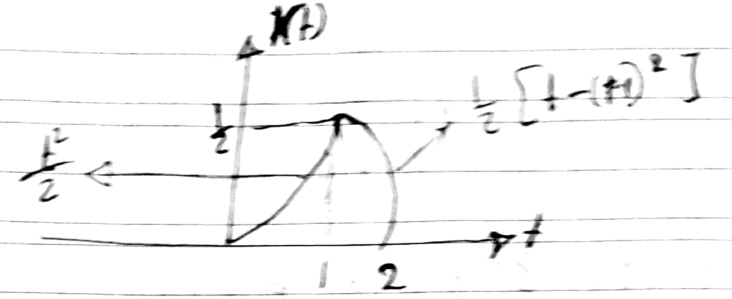
$y(t) = \frac{1}{2} \left[1 - (t-1)^2 \right]$

Case(4)



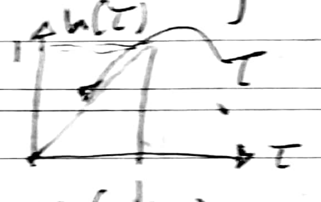
$t-1 > 1 \rightarrow t > 2$

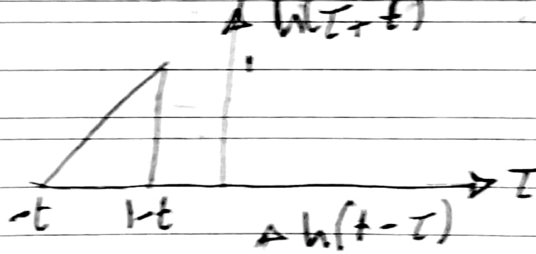
$y(t) = 0$

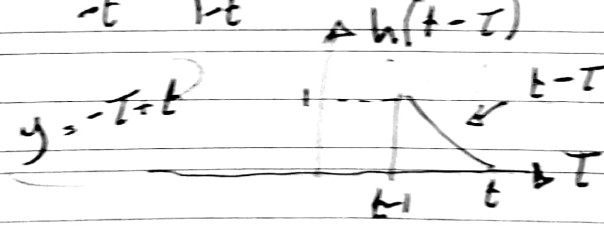


Ex

$$y(t) = x(\tau) * h(\tau) = \int x(\tau)\, h(t-\tau)\, d\tau$$





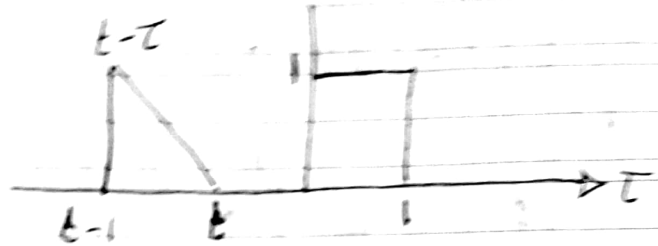


$$m = \frac{1}{t-(t-1)} = -1$$

$y = m\tau + c$

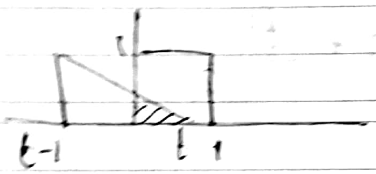
$0 = (-1)(\tau) + c$

$c = \tau$



Case (i) : $t < 0$

$y(t) = 0$

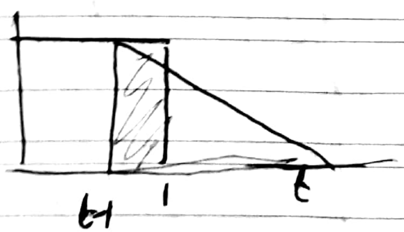


$0 < t < 1$ & $t-1 < 0$

$$y(t) = \int_0^t (t-\tau)(1)\, d\tau$$

$$= \left[t\tau - \frac{\tau^2}{2} \right]_0^t$$

$$y(t) = t^2 - \frac{t^2}{2} = \frac{t^2}{2}$$



$0 < t-1 < 1$ &

$t > 1$

$1 < t < 2$

$$y(t) = \int_{t-1}^{1} (t-\tau)(1)\, d\tau$$

$$= \left[t\tau - \frac{\tau^2}{2} \right]_{t-1}^{1}$$

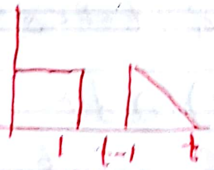
$$= \left[t - \frac{1}{2} \right] - \left[t(t-1) - \frac{(t-1)^2}{2} \right]$$

$$= \frac{2t-1}{2} - t^2 + t + \frac{t^2-2t+1}{2}$$

$$= -\frac{t^2}{2} + t$$

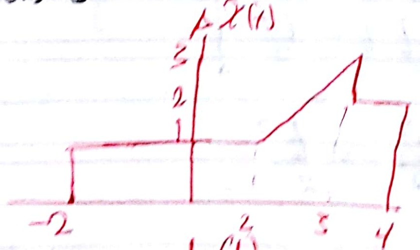
$t-1>1$

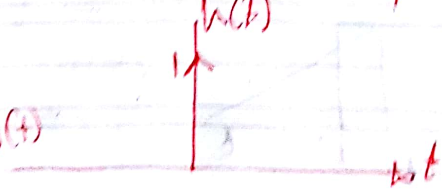
$t>2$



$y(t)=0$

Ex :





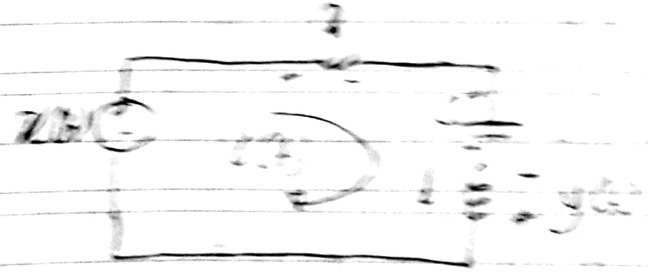
Find $y(t)$

$x(t) * \delta(t) = x(t)$

$x(t) * \delta(t-t_1) = x(t-t_1)$

$x(t-t_1) * \delta(t-t_2) = x(t-t_1-t_2)$

Ex. For the following ckt find the impulse response $h(t)$



$h(t) = y(t)$ when $x(t) = \delta(t)$

$-x(t) + R\,i(t) + \frac{1}{C}\int i(t)\,dt + L\frac{di(t)}{dt} = 0$

$$+L\frac{di(t)}{dt} + \frac{1}{c}\int i(t)\,dt + Ri(t) = x(t)$$

$$y(t) = +L\frac{di(t)}{dt} \rightarrow i(t) = -\frac{1}{L}\int y(t)\,dt$$

$$L\frac{d}{dt}\left(-\frac{1}{L}\int y(t)\,dt\right) + \frac{1}{c}\int -\frac{1}{L}\int y(t)\,dt$$

$$+R\left(\frac{1}{L}\int y(t)\,dt\right) = x(t)$$

$$-y(t) - \frac{1}{CL}\iint y(t)\,dt - \frac{R}{L}\int y(t)\,dt = x(t)$$

$$-\frac{d^2y(t)}{dt^2} - \frac{1}{LC}y(t) - \frac{R}{L}\frac{dy(t)}{dt} = \frac{d^2x(t)}{dt^2}$$

$$y''(t) + \frac{R}{L}y'(t) + \frac{1}{LC}y(t) = -x''(t)$$

$$h''(t) + \frac{R}{L}h'(t) + \frac{1}{LC}h(t) = -\frac{d^2}{dt^2}\delta(t)$$

For Zero input response :-

$$h''(t) + \frac{R}{L}h'(t) + \frac{1}{LC}h(t) = 0$$

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$R = 5\,\Omega \,,\ L = 1\,H \,,\ C = \frac{1}{3}\,F$

$h''(t) + 5h'(t) + 3h(t) = 0$

$m^2 + 5m + 3 = 0$

$m = \dfrac{-5 \pm \sqrt{25-(4)(1)(3)}}{2(1)}$ مميز

$m = \dfrac{-5 \pm \sqrt{13}}{2}$

$m_1 = -5+\sqrt{13}$ $m_2 = -5-\sqrt{13}$

$$m = \frac{-5 \pm \sqrt{3}\, i}{2}$$

$$m_1 = \frac{-5 + \sqrt{3}\, i}{2} \quad , \quad m_2 = \frac{-5 - \sqrt{3}\, i}{2}$$

$$h(t) = C_1 e^{m_1 t} + C_2 e^{m_2 t}$$

$$h(t) = C_1 e^{\left(-\frac{5}{2} + \frac{\sqrt{3}}{2} i\right)t} + C_2 e^{\left(-\frac{5}{2} - \frac{\sqrt{3}}{2} i\right)t}$$

$$h(t) = C_1 e^{-\frac{5}{2}t} \cos\left(\frac{\sqrt{3}}{2}t\right) + C_2 e^{-\frac{5}{2}t} \cos\left(\frac{\sqrt{3}}{2}t\right)$$

or

Sin

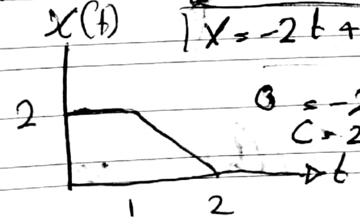
$$h(0^-) = 0$$

$$h'(0^-) = 1$$

Ex: Find $y(t)$

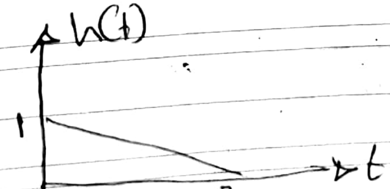
$$\boxed{x(t) = -2t + 4}$$

$$x = -2t + c$$



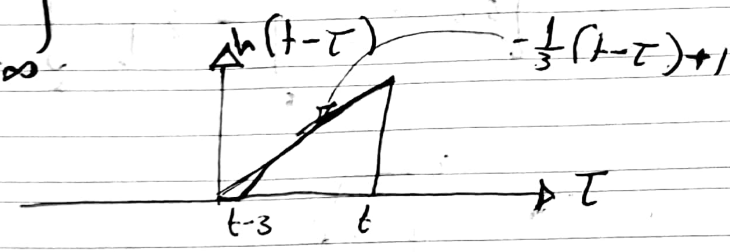
$$0 = -2(2) + c$$

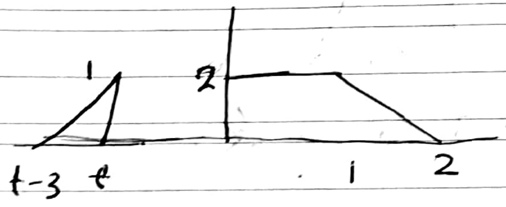
$$c = 2$$



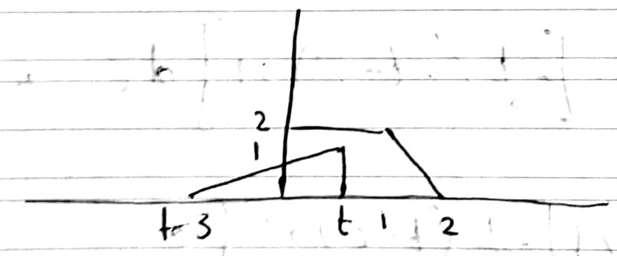
$$y(t) = x(t) * h(t)$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$





$t < 0 \quad y(t) = 0$



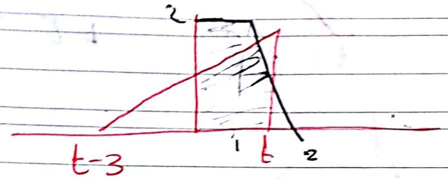
$0 < t < 1$ & $t-3 < 0 \rightarrow t < 3$

$$y(t) = \int_0^t (2)\left[-\frac{1}{3}(t-\tau)+1\right] d\tau$$

$$= \frac{2}{3}\int_0^t (t-\tau)\, d\tau + 2\int_0^t d\tau$$

$$= \frac{2}{3}\left[t\tau - \frac{\tau^2}{2}\right]_0^t + 2\left[\tau\right]_0^t$$

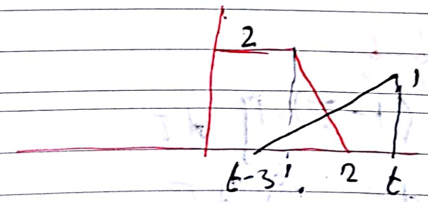
كمل تكاملها



$1 < t < 2$ & $t-3 < 0 \rightarrow t < 3$

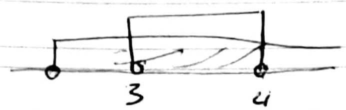
$$y(t) = \int_0^1 (2)\left[-\frac{1}{3}\left(-t-\tau\right)+1\right]d\tau \;+$$

$$\int_1^t \left[-2\tau+4\right]\left[-\frac{1}{3}\left(t-\tau\right)+1\right]d\tau$$



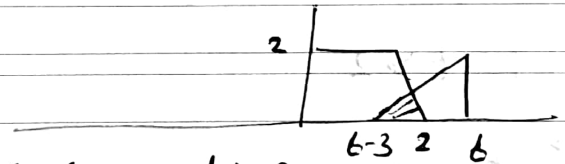
$0 < t-3 < 1$ & $t > 2$

$3 > t < 4$ $\quad t > 2$



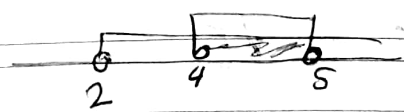
$$y(t) = \int_{t-3}^{1} (2)\left[-\frac{1}{3}(t-\tau)+1\right] d\tau +$$

$$\int_{1}^{2} (-2\tau+4)\left[-\frac{1}{3}(t-\tau)+1\right] d\tau$$

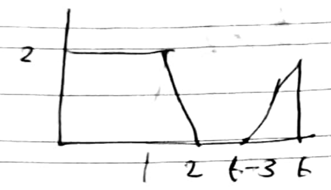


$1 < t-3 < 2$ & $t > 2$

$4 < t < 5$ & $t > 2$



$$y(t) = \int_{t-3}^{2} (\ )(\ )\ d\tau$$



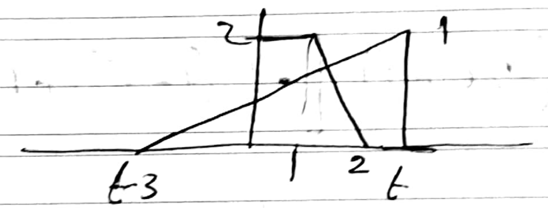
$t-3>2$

$t>5$

$y(t)$

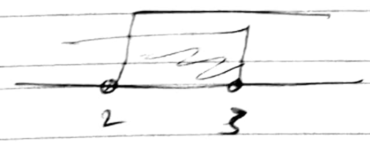
op:iv

$2<t<3$



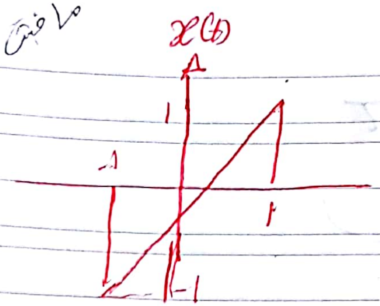
$t-3<0$ & $t>2$

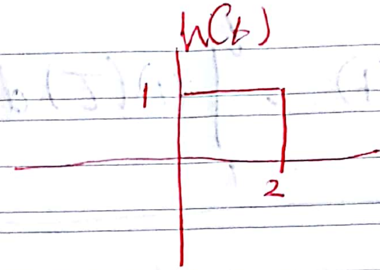
$t<3$ & $t>2$



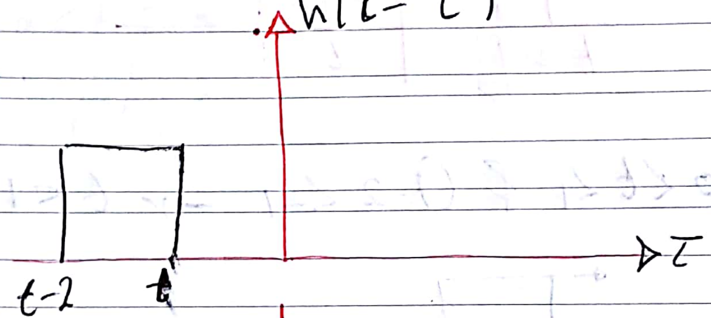
$y(t) = \int_0^1 (\ )(\ )\,d\tau + \int_1^2 (\ )(\ )\,d\tau$

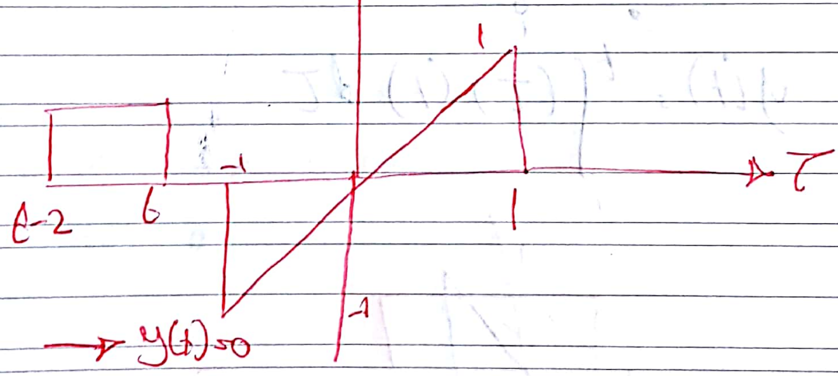
ما فهمت بس كتبتها



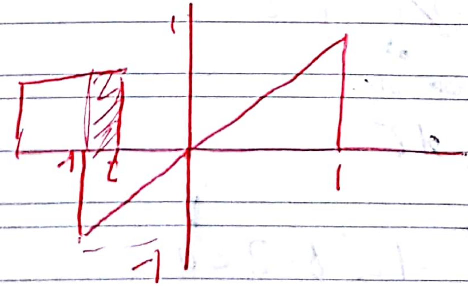


$y(t) = x(t) * h(t)$



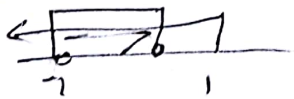


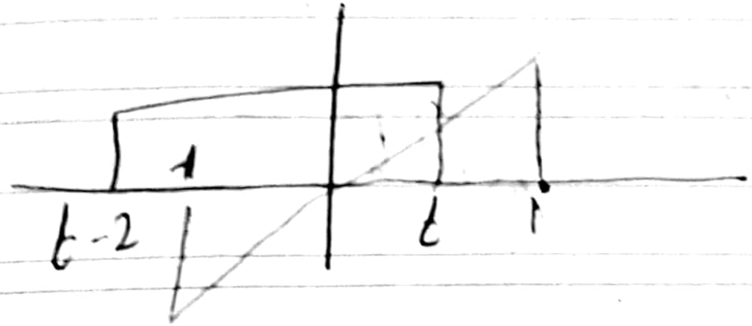
$t < -1 \longrightarrow y(t) = 0$



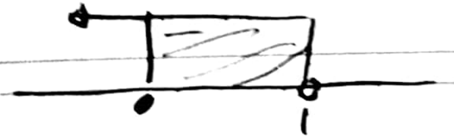
$-1 < t < 0$ &

$t - 2 < -1$

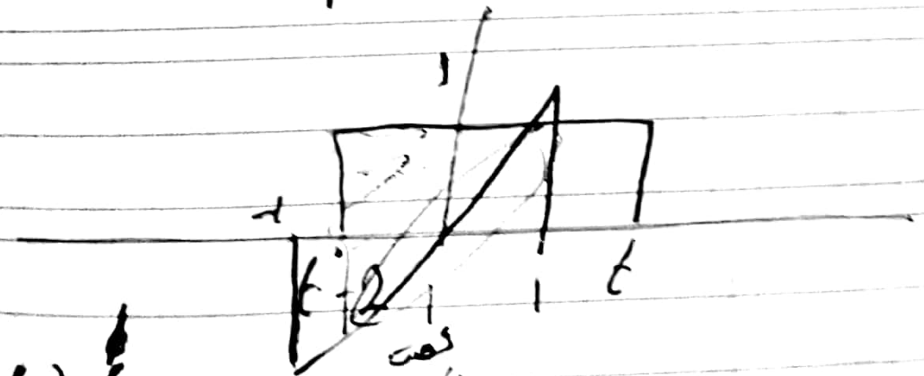




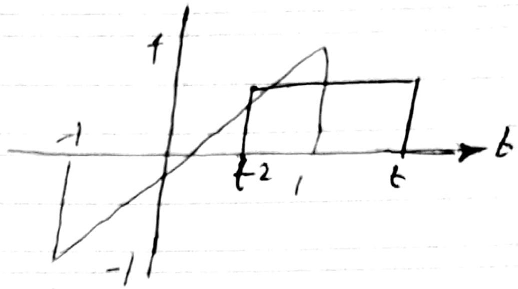
$0 < t < 1$ & $(t-2 < -1 \rightarrow t < 1$



$$y(t) = \int_{1}^{t} (\tau)(1)\, d\tau$$

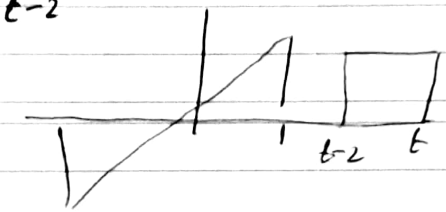


$$y(t) \int_{t-2} (\tau)(1)\, d\tau$$



$t > 1$ & $0 < t-2 < 1$

$$y(t) = \int_{t-2}^{1} (\tau)(1)\, d\tau$$



$t-2 > 1 \rightarrow t > 3$

$y(t) = 0$